CHAPTER IV

THEORETICAL CALCULATION OF SHOCKFORCES

ABSTRACT

The shock pressure that can develop when a mass of water hits a rigid plane construction (like a breaking wave hitting a vertical wall) is calculated theoretically and the results are presented in a graph. The maximum pressure depends on the velocity of the oncoming water and its content of airbubbles. It does not depend on the size of the water. When the water approaches the construction some of the air in between is forced out, whereby it delivers a reactive force of important magnitude, in spite of the low density of the air. This reactive force is balancing the compression that takes place for the rest of the air. The maximum pressure is then reached before the water has touched the wall. As the shock process is different in the model case and in the prototype case, the results do not fit into Froude's model scale law, as can easily be seen on the graph.

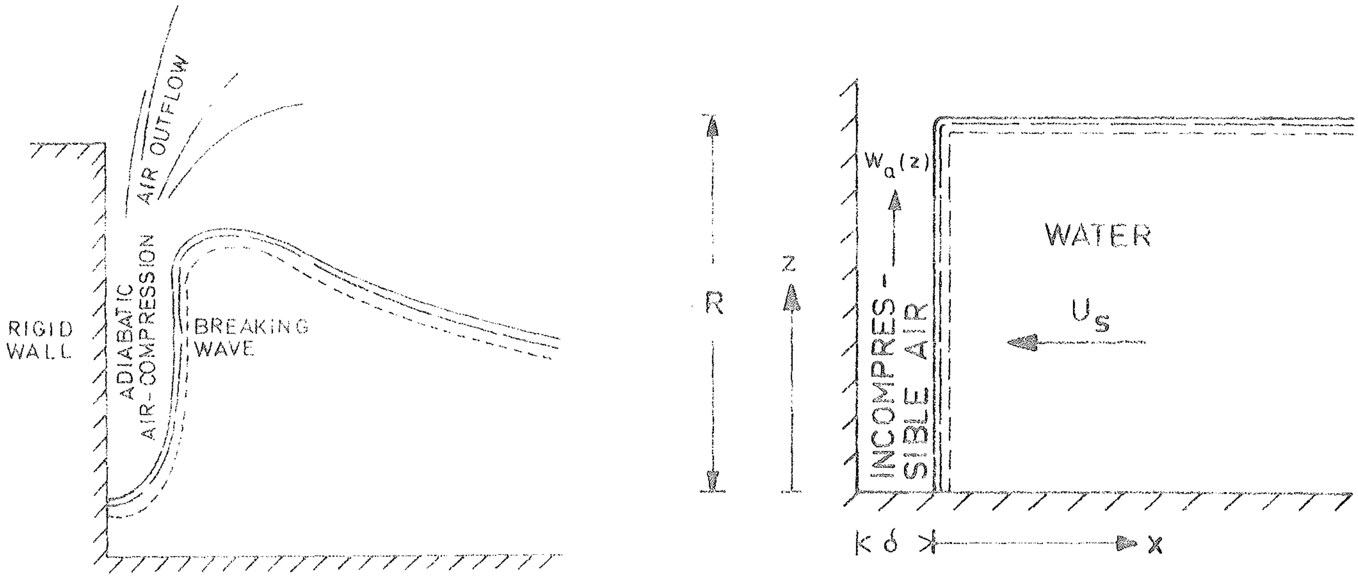

Fig.1. Introduction and definition sketch

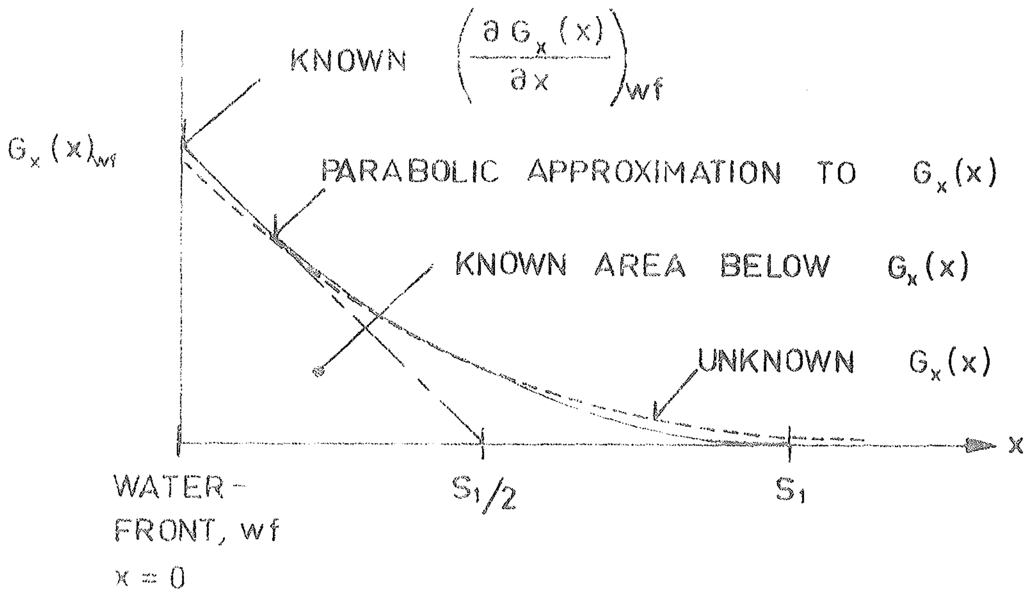

Fig.2. Graph of horizontal particle acceleration

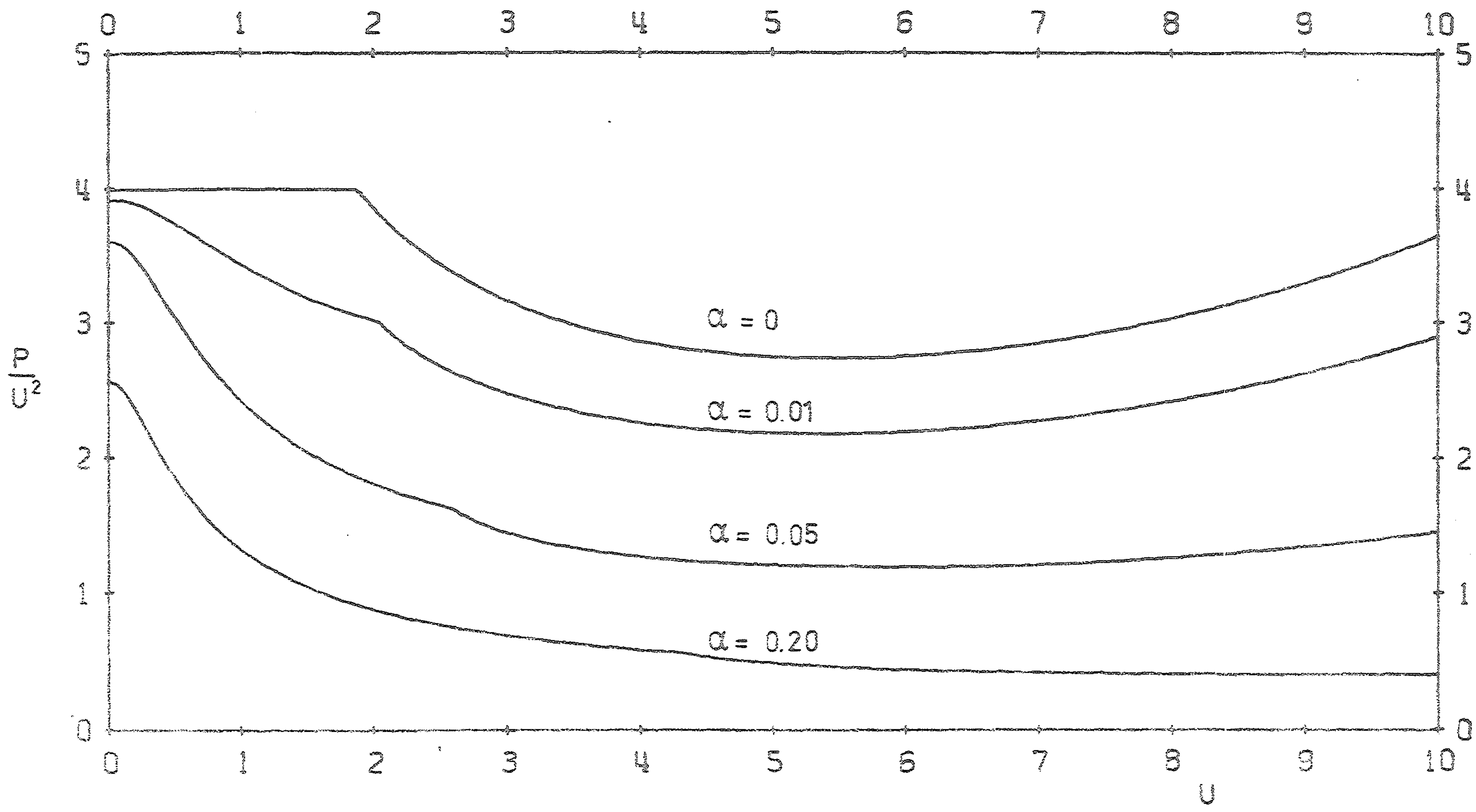

Fig.3. The maximum shock pressure in the plane case in metric units

INTRODUCTION

From observations of nature and model tests it is found plausible that a wave can approach a rigid wall in a dangerous manner like the one shown on fig.1 left, giving a slit of air between the wall and the waterfront. The air starts a balanced adiabatic compression and outflow, and the calculations show that the compressibility of this air and the very modest density of the air in the outflow are the physical factors governing the shock process, and that the compressibility of the water is negligible.

The course of the shock can with good approximation be divided into two physical distinct cases. First, when the pressure is still low, case I, the air is regarded incompressible and the pressure is given by the reactive force of the air outflow. Second, when the water has moved closer to the wall and the pressure is getting higher, case II, the outflow and adiabatic compression will be substituted by no outflow and isothermal compression.

Although the formulas are evaluated by looking at the unrealistic case, fig.1 right, the results show greater usefulness, and water with airbubbles and a rough waterfront is treated. The difficulties in deciding the velocity of the waterfront etc. of a breaking wave is not touched upon.

Except where otherwise mentioned all the considerations are for a plane case.

All non-dimension-homogenious equations are given in the technical unit system, in megapond, Mp, and metres, m, and seconds, s. For the density of the water is used the fresh water term, $\varrho_v = 0.102$ Mp $\cdot$ S^2/m^4.

CASE I. INCOMPRESSIBLE AIR

The idealized case where a mass of water with plane vertical front proceeds towards a plane vertical wall as shown on fig.1 right will first be considered. The horizontal velocity U_s of the water is provisionally regarded as a constant over the time, t, and over z (commented after (25)). By this the air between the water and the wall is forced out with a velocity $W_a(z)$. The kinetic energy of the outflow is lost, so that the pressure $p(R) = 0$.

As the air for the present is assumed incompressible the usual equation of continuity holds:

$$U_s \cdot z = \delta \cdot W_a(z) \tag{1}$$

where δ is the thickness of the air slit. Remembering that δ is timedependent the acceleration of an air-particle is deduced from (1):

$$\frac{dW_a(z)}{dt} = \frac{\partial W_a(z)}{\partial t} + W_a(z) \cdot \frac{\partial W_a(z)}{\partial z} = 2 \cdot \frac{U_s^2}{\delta^2} \cdot z \tag{2}$$

Neglecting the gravitational force the vertical equilibrium of an air-particle yields the pressure $p(z)$ above atmospheric pressure of the air in front of the water of:

$$\frac{\partial p(z)}{\partial z} = -\varrho_a \frac{dW_a(z)}{dt} = -2\varrho_a \frac{U_s^2}{\delta^2} \cdot z \tag{3}$$

where ϱ_a is the density of the air at atmospheric pressure, p_{atm}.

Integrating and using $p(R) = 0$:

$$p(z) = \varrho_a \frac{U_s^2}{\delta^2} (R^2 - z^2) \tag{4}$$

The mean value over z becomes:

$$p^* = \frac{2}{3}\varrho_a \cdot \frac{U_s^2}{\delta^2} \cdot R^2 \tag{5}$$

For δ close to 0 this formula yields a fast rise in pressure. With the absolute pressure being $p_{abs} = p(z) + p_{atm}$, an adiabatic compression of a confined quantity of air would only yield :

$$p_{abs\ a} = \text{constant} \cdot \frac{1}{\delta^{1,4}}$$

and an isothermal compression:

$$p_{abs\ i} = \text{constant} \cdot \frac{1}{\delta} = k_i \cdot \frac{1}{\delta} \tag{6}$$

TRANSITION TO CASE II

Differentiating the isothermal expression (6):

$$\frac{dp_{abs\ i}}{d\delta} = - k_i \frac{1}{\delta^2} = - \frac{p_{abs\ i}}{\delta} \tag{7}$$

With the absolute pressure in (4) being $p_{abs\ u} = p(z) + p_{atm}$ and differentiating it:

$$\frac{dp_{abs\ u}}{d\delta} = -2 \cdot \varrho_a \cdot \frac{U_s^2}{\delta^3}(R^2 - z^2) = -2\,(p_{abs\ u} - p_{atm}) \cdot \frac{1}{\delta} \qquad (8)$$

By suddenly changing from the outflow formula (4) to the isothermal formula (6) at $\delta = \delta_I$, where then $p_{abs\ u} = p_{abs\ i}$, the rise in pressure will be less for:

$$\left|\frac{dp_{abs\ i}}{d\delta}\right| < \left|\frac{dp_{abs\ u}}{d\delta}\right| \qquad (9)$$

$$2\,(1 - \frac{p_{atm}}{p_{abs\ u}}) > 1$$

$$p_{abs\ u} > 2\ p_{atm} \qquad (10)$$

$$p(z) > p_{atm}$$

Using the mean-pressure (5) δ_I is found to:

$$p^* = \frac{2}{3}\varrho_a \cdot \frac{U_s^2}{\delta_I^2} \cdot R^2 = p_{atm}$$

$$\delta_I = \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{\varrho_a}{p_{atm}}} \cdot U_s \cdot R = 2.9 \cdot 10^{-3} \cdot U_s \cdot R\ (m) \qquad (11)$$

To create the compression of the air that actually has taken place up to δ_I with $p_{abs} = 2\ p_{atm}$ is needed a little additional air. As seen after (50) there will be supplied some extra.

For $\delta < \delta_I$ the compression will be far greater than incompressibility. Because of sufficient thickness of the airlayer, and the short duration of the shock the compression is adiabatic. This compression combined with the outflow will in case II be treated as an isothermal compression after formula (6). Further reasons for this choice are given after (50). So in case II the mean-pressure is:

$$p^* = \frac{2 \cdot p_{atm} \cdot \delta_I}{\delta} - p_{atm} \tag{12}$$

When a pressure-distribution is needed it will be chosen as in (4):

$$p(z) = \frac{3}{2} \cdot p^* \cdot (1 - \frac{z^2}{R^2}) \tag{13}$$

HYDRODYNAMIC MASS

Using (3) on water by changing ϱ_a to ϱ_v, the density of water, and by (5) and (13) the vertical acceleration of the water in the waterfront is found to:

$$G_z(z)_{wf} = -\frac{1}{\varrho_v}\frac{\partial p(z)}{\partial z} = 3 \cdot \frac{1}{\varrho_v} \cdot p^* \frac{z}{R^2} \tag{14}$$

A co-ordinatesystem (x,z) moves with U_s. u and w being the horizontal and vertical velocity respectively, at the place regarded, the acceleration terms $G_x(x)$ and $G_z(z)$ will be:

$$G_x(x) = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial z} \cdot w$$

$$G_z(z) = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \cdot u + \frac{\partial w}{\partial z} \cdot w$$

$$\frac{\partial G_x(x)}{\partial x} = \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial x^2} \cdot u + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial x \partial z} \cdot w + \frac{\partial u}{\partial z} \cdot \frac{\partial w}{\partial x} \qquad (15)$$

$$\frac{\partial G_z(z)}{\partial z} = \frac{\partial^2 w}{\partial z \partial t} + \frac{\partial^2 w}{\partial x \partial z} \cdot u + \frac{\partial w}{\partial x} \cdot \frac{\partial u}{\partial z} + \frac{\partial^2 w}{\partial z^2} \cdot w + \left(\frac{\partial w}{\partial z}\right)^2$$

Regarding the waterfront where the timedependent changes are so rapid it is justified to retain only the first term in $\frac{\partial G_x(x)}{\partial x}$ and $\frac{\partial G_z(z)}{\partial z}$. After (22) it is shown that $\frac{\partial u}{\partial x}$ is comparatively small. By the equation of continuity,

$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$, it is got:

$$\left(\frac{\partial G_x(x)}{\partial x}\right)_{wf} + \left(\frac{\partial G_z(z)}{\partial z}\right)_{wf} = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0 \qquad (16)$$

Then by differentiating (14):

$$-\left(\frac{\partial G_x(x)}{\partial x}\right)_{wf} = \left(\frac{\partial G_z(z)}{\partial z}\right)_{wf} = 3 \cdot \frac{1}{\varrho_V} \cdot p^* \cdot \frac{1}{R^2} \qquad (17)$$

Further information about $G_x(x)$ can be given from the horizontal dynamic equation:

$$p(z) = \varrho_V \cdot \int_{wf}^{\infty} G_x(x)dx \qquad (18)$$

As shown on fig.2 $G_x(x)$ will be approximated by a parabola. A certain degree of inaccuracy in selecting the approximative function to $G_x(x)$ will give a lower degree of inaccuracy in the wanted result, namely $G_x(x)_{wf}$. For easier calculations x is put to 0 in the waterfront. By looking at fig.2 the length S_1 is found to:

$$S_1 = -2 \frac{G_x(x)_{wf}}{\left(\frac{\partial G_x(x)}{\partial x}\right)_{wf}} \tag{19}$$

Then from (18):

$$p(z) = \frac{1}{3} \varrho_v \cdot S_1 \cdot G_x(x)_{wf} = -\frac{2}{3} \varrho_v \cdot \frac{G_x(x)^2_{wf}}{\left(\frac{\partial G_x(x)}{\partial x}\right)_{wf}} \tag{20}$$

and from (17) and (13):

$$G_x(x)_{wf} = \sqrt{-\frac{3}{2} \cdot \frac{p(z)}{\varrho_v} \left(\frac{\partial G_x(x)}{\partial x}\right)_{wf}} = \frac{3\sqrt{3}}{2} \cdot \frac{1}{\varrho_v} \cdot p^* \cdot \frac{1}{R} \sqrt{1 - \frac{z^2}{R^2}} \tag{21}$$

The hydrodynamic mass will accelerate with the waterfront, $G_x(x)_{wf}$, when acted upon by its pressure $p(z)$. So it is possible to deduce a hydrodynamic mass-length, S, by using (20):

$$\varrho_v \cdot S \cdot G_x(x)_{wf} = p(z) = \frac{1}{3} \varrho_v \cdot S_1 \cdot G_x(x)_{wf}$$

Hereby S is derived from (19), (21) and (17):

$$S = \frac{1}{\sqrt{3}} \sqrt{R^2 - z^2} = 0.58 \cdot \sqrt{R^2 - z^2} \qquad (22)$$

This expression is in force for both case I and II as the pressure distribution is set to the same (13). But even if the pressure distribution is altered somewhat, it is seen from (13), (14), (17), (21) and (19) that S in (22) ends up being close to the same, although the intermediate results differ.

As the velocity u changes gradually over the distance S_1 it can be seen that $\frac{\partial u}{\partial x}$ is comparatively small in (15).

The mean value of S over z is:

$$S^* = \frac{\int_0^R \frac{1}{\sqrt{3}} \sqrt{R^2 - z^2}\, dz}{R} = \frac{\pi}{4\sqrt{3}} \cdot R = 0.45 \cdot R \qquad (23)$$

$$S^* = \frac{\pi}{4} \cdot S_{max}$$

where $S_{max} = S$ for $z = 0$.

CHANGE OF VELOCITIES

Using mean values of pressure (5) and hydrodynamic mass (23) the change in momentum, ΔI, of the water will be:

$$\varrho_v \cdot S^* \cdot \Delta U_I = \Delta I = \int p^* \cdot dt = \int_\infty^{\delta_I} - p^* \frac{1}{U} \cdot d\delta =$$

$$- \int_\infty^{\delta_I} \frac{2}{3} \varrho_a \cdot \frac{U}{\delta^2} R^2 \, d\delta \qquad (24)$$

where U is the momentary horizontal velocity of the water. Considering U independent of δ, $U = U_s$, the mean value of the change in horizontal velocity, ΔU_I, is, using (11) and (23):

$$\Delta U_I = \frac{2}{3} \frac{\varrho_a}{\varrho_v} \frac{U_s}{\delta_I} \frac{1}{S*} R^2 = \frac{4\sqrt{2}}{\pi} \cdot \frac{1}{\varrho_v} \sqrt{\varrho_a \cdot p_{atm}} = 0.65 \text{ m/s} \qquad (25)$$

For the design waves of the nature this reduction in the velocity, U is so small, that it was reasonable to regard it as a constant, $U = U_s$, above in (25) and in (1).

Instead of using $U = U_s$ in $dt = \frac{1}{U} d\delta$ in (24) it could have been reasonable to use a mean value, so $U = U_s - \frac{1}{2} \cdot \Delta U_I$, making ΔU_I depending on U_s. That is done in the graph, fig.3.

Using (14) and (5) the vertical velocity of the waterfront, V_I, for $\delta = \delta_I$ is found:

$$\frac{\partial V}{\partial t} \cong G_z(z)_{wf} = 2 \cdot \frac{\varrho_a}{\varrho_v} \cdot \frac{U_s^2}{\delta^2} \cdot z$$

$$V_I = \int G_z(z)_{wf} \cdot dt = -\int_{\infty}^{\delta_I} G_z(z)_{wf} \cdot \frac{1}{U} \cdot d\delta$$

$$= 2 \cdot \frac{\varrho_a}{\varrho_v} \cdot \frac{U_s}{\delta_I} \cdot z = \sqrt{6} \cdot \frac{1}{\varrho_v} \sqrt{\varrho_a \cdot p_{atm}} \cdot \frac{z}{R}$$

$$= 0.89 \cdot \frac{z}{R} \quad \text{m/s} \qquad (26)$$

The hydrodynamic mass (23), can not be used quite as usually in the short duration of the impulse, because when the waterfront has stopped, the back of S^* is able to travel with that horizontal velocity, ΔU_{b1} that is allowed by the vertical velocity through the equation of continuity.

Instead of using the real horizontal distribution of the vertical velocity, V_I from the waterfront is used with $z = R$, $V_{I\ top}$, but only over the length of S^*. In this way ΔU_{b1} will be:

$$\Delta U_{b1} = \frac{V_{I\ top} \cdot S^*}{R} = \frac{\pi}{2\sqrt{2}} \frac{1}{\varrho_v} \cdot \sqrt{\varrho_a \cdot p_{atm}} = 0.40 \text{ m/s} \qquad (27)$$

So S^* can with stopped waterfront have a mean horizontal velocity of $\frac{\Delta U_{b1}}{2}$. Compared with ΔU_I in (25) that reveals:

$$\frac{\Delta U_{b1}/2}{\Delta U_I} = \frac{\pi^2}{32} = 0.31 \qquad (28)$$

When $\delta = \delta_I$ the impulsegiving horizontal velocity of the waterfront is reduced from U_s to:

$$U_I = U_s - \Delta U_I - \frac{\Delta U_{b1}}{2} - \Delta U_{Iab} = U_s - 0.85 - \Delta U_{Iab} \text{ (m/s)} \qquad (29)$$

where $\Delta U_{Iab} = 0$ when there are no airbubbles in the water. (see (56))

CASE II. COMPRESSIBLE AIR.

Now the water gets closer to the wall than the small distance δ_I. The calculations will again be performed for mean values, so the pressure is as announced in (12):

$$p^* = \frac{2 \cdot p_{atm} \cdot \delta_I}{\delta} - p_{atm}$$

and the problem is to stop the velocity U_I (29) of S^* (23). Moving from δ_I to δ against the pressure p^*, the water performs a work of:

$$E_p = \int_{\delta_I}^{\delta} - p^* \cdot d\delta = 2p_{atm} \cdot \delta_I \cdot \ln \frac{\delta_I}{\delta} - p_{atm} (\delta_I - \delta)$$

From the horizontal velocity the water has a kinetic energy of:

$$E_k = \frac{1}{2} \varrho_v \cdot S^* \cdot U_I^2$$

Stopping U_I a vertical velocity V_2 will be generated like in (26). If the vertical velocities could be neglected the principles of energy could be used, whereby the impulse would be stopped for $\delta = \delta_{min}$ by using $E_k = E_p$:

$$\frac{1}{2} \varrho_v \cdot S^* \cdot U_I^2 = 2p_{atm} \cdot \delta_I \cdot \ln \frac{\delta_I}{\delta_{min}} - p_{atm} (\delta_I - \delta_{min}) \quad (30)$$

Dividing over by U_I and changing the left side U_I to ΔU_2 (30) is changed to an equation of momentum:

$$\varrho_v \cdot S^* \cdot \Delta U_2 = \frac{2}{U_I}\left[2 \cdot p_{atm} \cdot \delta_I \cdot \ln\frac{\delta_I}{\delta_{min}} - p_{atm}(\delta_I - \delta_{min})\right] \tag{31}$$

and by using S^* from (23):

$$\Delta U_2 = \frac{8\sqrt{3}}{\pi} \cdot \frac{1}{\varrho_v} \cdot p_{atm} \cdot \delta_I \cdot \frac{1}{R} \cdot \frac{1}{U_I} \cdot \left[2\ln\frac{\delta_I}{\delta_{min}} + \frac{\delta_{min}}{\delta_I} - 1\right] \tag{32}$$

Going from (30) to (31) the timedependent velocity distribution in the equation of momentum, (31), actually was decided to be so, that the mean value of U is $\frac{U_I}{2}$ and thereby:

$$dt = -\frac{2}{U_I} \cdot d\delta \tag{33}$$

Because the forces involved in braking the horizontal velocity and generating the vertical are the same at any moment, V_2 will be a constant proportion of ΔU_2 (see (35)). Therefore the principles of energy are appliable using only the horizontal velocity, when the involved mass, S^*, is reduced correspondingly to the effect of the vertical velocities. So (33) is in force here in the general case making (32) appliable when consideration to V_2 is done in the usual manner (like in (29)) in ΔU_2.

The generated vertical velocity is then got from (14) and (33):

$$V_2 = \int G_z(z)_{wf}\, dt = -\int_{\delta_I}^{\delta_{min}} 3 \cdot \frac{1}{\varrho_v} \frac{p^*}{R^2} \cdot z \cdot \frac{2}{U_I}\, d\delta =$$

$$6 \cdot \frac{1}{\varrho_v} \cdot \frac{1}{U_I} \cdot \frac{z}{R^2} \int_{\delta_{min}}^{\delta_I} \left(\frac{2p_{atm} \cdot \delta_I}{\delta} - p_{atm} \right) d\delta =$$

$$6 \cdot \frac{1}{\varrho_v} \cdot p_{atm} \cdot \delta_I \cdot \frac{1}{U_I} \cdot \frac{z}{R^2} \left[2 \ln \frac{\delta_I}{\delta_{min}} + \frac{\delta_{min}}{\delta_I} - 1 \right] \tag{34}$$

With $V_{2\ top} = V_2$ for $z = R$ (34) and (32) are compared:

$$\frac{V_{2\ top}}{\Delta U_2} = \frac{\sqrt{3}\pi}{4} = 1.4 \tag{35}$$

Like in (27) this makes a difference, ΔU_{b2} , in front and back velocity of S^* :

$$\Delta U_{b2} = \frac{V_{2\ top} \cdot S^*}{R} = \frac{\pi}{4\sqrt{3}} \cdot V_{2\ top} = 0.45 \cdot V_{2\ top}$$

$$\frac{\Delta U_{b2}}{2} = \frac{\pi^2}{32} \cdot \Delta U_2 = 0.31 \cdot \Delta U_2 \tag{36}$$

The impulsegiving velocity is then:

$$\Delta U_2 = U_I - \frac{\Delta U_{b2}}{2} = U_I - \frac{\pi^2}{32} \cdot \Delta U_2$$

$$\Delta U_2 = \frac{1}{1 + \frac{\pi^2}{32}} \cdot U_I = 0.76 \cdot U_I \tag{37}$$

Substituting δ_I with (11) the equation of momentum (32) now yields:

$$U_I = \beta \cdot \left(1+\frac{\pi^2}{32}\right) \cdot \frac{8\sqrt{2}}{\pi} \cdot \frac{1}{\varrho_v} \cdot \sqrt{\varrho_a \cdot p_{atm}} \cdot \frac{U_s}{U_I}$$

$$\cdot \left[2\ln\frac{\delta_I}{\delta_{min}} + \frac{\delta_{min}}{\delta_I} - 1\right] = 1.71 \cdot \beta \cdot \frac{U_s}{U_I} \cdot \left[2\ln\frac{\delta_I}{\delta_{min}}\right.$$

$$\left. + \frac{\delta_{min}}{\delta_I} - 1\right] \text{ m/s} \qquad (38)$$

where $\beta = 1$, when there is no content of airbubbles in the water as hitherto.

Finally (12) is used to get the wanted maximum mean pressure:

$$p^*_{max} = 2 \cdot p_{atm} \cdot \frac{\delta_I}{\delta_{min}} - p_{atm} = 20.7 \cdot \frac{\delta_I}{\delta_{min}} - 10.3 \text{ Mp/m}^2 \qquad (39)$$

MODEL CASE

With low start values of the horizontal velocity, U_s , the impulse is over before case II is reached. So U can not be considered constant as in (25). Using (33) which is also in force here, the change in horizontal momentum, $\Delta I(z)$, is deduced from the general expressions (22) and (4):

$$\varrho_v \cdot s \cdot \Delta U_m = \Delta I(z) = \int p(z) \cdot dt = -\int_{\infty}^{\delta_{min}} p(z) \; \frac{2}{U_s} \cdot d\delta$$

$$\Delta U_m = 2 \cdot \frac{\varrho_a}{\varrho_v} \cdot \frac{1}{S} \cdot \frac{U_s}{\delta_{min}} \cdot (R^2 - z^2) \tag{40}$$

where as in (29):

$$\Delta U_m = U_s - \frac{\Delta U_{bm}}{2} \tag{41}$$

From (40) and (22):

$$\delta_{min}(z) = 2\sqrt{3} \cdot \frac{\varrho_a}{\varrho_v} \cdot \sqrt{R^2 - z^2} \cdot \frac{U_s}{\Delta U_m}$$

$$= 4.5 \cdot 10^{-3} \cdot \sqrt{R^2 - z^2} \cdot \frac{U_s}{\Delta U_m} \tag{42}$$

For $z \to R$, $\delta_{min}(z) \to 0$, so there is a tendency to closure of the air outlet.

The mean value of $\delta_{min}(z)$ (42) over z is:

$$\delta_{min} = \frac{\pi}{4} \cdot 2\sqrt{3} \cdot \frac{\varrho_a}{\varrho_v} \cdot R \cdot \frac{U_s}{\Delta U_m} = 3.5 \cdot 10^{-3} \cdot R \cdot \frac{U_s}{\Delta U_m} \tag{43}$$

Like in (26) the generated vertical velocity for $z = R$, $V_{m\ top}$, is got from (33), (14), (5) and (43):

$$V_{m\ top} = \int G_z(z)_{wf} \cdot dt = -\int_{\infty}^{\delta_{min}} G_z(z)_{wf} \frac{2}{U_s} d\delta$$

$$= 4 \cdot \frac{\varrho_a}{\varrho_v} \cdot \frac{U_s}{\delta_{min}} \cdot R = \frac{8}{\pi\sqrt{3}} \cdot \Delta U_m = 1.47 \cdot \Delta U_m \tag{44}$$

Just as in (27), (23) then yields:

$$\frac{\Delta U_{bm}}{2} = \frac{1}{2} \cdot \frac{V_{I\ top} \cdot S^*}{R} = \frac{1}{3} \cdot \Delta U \tag{45}$$

Now (41) yields:

$$\Delta U_m = \frac{3}{4} \cdot U_s - \Delta U_{mab} \tag{46}$$

where $\Delta U_{mab} = 0$ when there are no airbubbles in the water, (see(57)). The maximum mean pressure is hereby got from (5) and (43):

$$p^*_{max} = \frac{8}{9\pi^2} \frac{\varrho_v^2}{\varrho_a} \cdot \Delta U_m^2 = 7.1 \cdot \Delta U_m^2 \text{ (Mp/m}^2\text{)} \tag{47}$$

With $\Delta U_{mab} = 0$ (47) will be:

$$p^*_{max} = \frac{1}{2\pi^2} \frac{\varrho_v^2}{\varrho_a} \cdot U_s^2 = 4.0 \cdot U_s^2 \text{ (Mp/m}^2\text{)} \tag{48}$$

For $U_s = 1.9$ m/s the pressure found by (48) is the same as found by (39).

DURATION OF THE SHOCK

The rise and fall of the pressure will be viewed as symmetrical although it is problematic if it is (the air outflow shortens the fall, the behind the waterfront approaching water makes it longer). The duration of the shock, $\Delta\tau$, for the prototype will then be defined as the time the pressure is more than p_{atm} . Using (11):

$$\Delta\tau = 2 \cdot \frac{\delta_I}{U_s/2} = 4\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{\varrho_a}{p_{atm}}} \cdot R = 1.17 \cdot 10^{-2} \cdot R \text{ (s)} \tag{49}$$

In the model case $\Delta\tau$ is defined as the time the pressure is greater than $\frac{1}{16}$ of the maximum pressure. That happens for $\delta < 4 \cdot \delta_{min}$ (43) whereby:

$$\Delta\tau = 2 \cdot \frac{4\ \delta_{min} - \delta_{min}}{U_s/2} = 12 \cdot \frac{\delta_{min}}{U_s} \tag{50}$$

CONSIDERATION OF MAIN ASSUMPTIONS

Using the assumption that the water all the time during the shock moves as shown on fig.1 right a more accurate calculation can be done. Because of compression of the air it will show a deficiency in pressure at the end of case I and the beginning of case II, if there is allowed the big outflow as $W(R)$ in (1) gives. But as (42) shows, the outlet is narrowed, reducing the outflow. This again promotes a more uniform vertical pressure distribution making the mean pressure usable from $z = 0$ to almost $z = R$. For ocean size waves the pressure at the end of case II gets higher than calculated because of the adiabatic compression. But as the air gets more compressed its expansion is first finished outside the outlet, making the pressure in the outlet greater than 0. This widens the outlet allowing more air to escape.

If the influence of the air in the slit was disregarded and the pressure from compressible water then calculated, it would reveal about 10 times as high pressures. Therefore the compressibility of the water can be disregarded.

From (35) and (11) it can be seen that R changes only about 1% in size before maximum pressure is reached.

AIRBUBBLES AND UNSMOOTH WATERFRONT

Airbubbles in the oncoming water near the waterfront will soften the shock because they will get compressed. For bubbles up to centimeter size the compression will be isothermal, so it is possible to use the same formula as in front of the water, (6). With $G_x(x)$ proportional to x^2 (see fig.2), the horizontal pressure distribution $p(x_1)$ will be:

$$p(x_1) = p^* \cdot x_1^3$$

where $x_1 = 1 - \frac{x}{3S^*}$

The volume of airbubbles within a unit cube at x is called $\theta_o(x)$ when there is no pressure, and $\theta_p(x)$ when the pressure is $p(x_1)$. The isothermal compression law then yields:

$$\frac{\theta_p(x)}{\theta_o(x)} = \frac{p_{atm}}{p(x_1) + p_{atm}} = \frac{p_{atm}}{p^* \cdot x_1^3 + p_{atm}}$$

This gives a reduction, $\Delta\theta(x)$, in the volume of airbubbles:

$$\frac{\Delta\theta(x)}{\theta_o(x)} = 1 - \frac{\theta_p(x)}{\theta_o(x)} = \frac{p^* \cdot x_1^3}{p^* \cdot x_1^3 + p_{atm}}$$

For practical use $\frac{\Delta\theta(x)}{\theta_o(x)}$ can be taken as parabolic in x, as that agrees well for $\frac{1}{3}\, p_{atm} < p^* < 2\, p_{atm}$. With an evenly distributed airbubble content, the relative amount at $p^* = 0$ being α, the total reduction, $\Delta\theta_{tot}$, in bubble volume by compression at p^* therefore is the same as was $\Delta\theta(x)$ from the waterfront valid over S^*. So:

$$\Delta\theta_{tot} = \alpha \cdot S^* \cdot \frac{p^*}{p^* + p_{atm}} \tag{51}$$

This $\Delta\theta_{tot}$ gives a further distance to stop the impulse of the water.

For ocean size U_s the calculation as usual is divided into case I and case II. For $p^* = p_{atm}$ (51) will be:

$$\Delta\theta_{tot\ I} = \frac{1}{2} \cdot \alpha \cdot S^*$$

In deciding the change in horizontal velocity, ΔU_{Ia} from the airbubbles there is needed:

$$\int_{p^*=0}^{p^*=p_{atm}} p^* \cdot d\,\Delta\theta_{tot} = (\ln 2 - \frac{1}{2}) \cdot p_{atm} \cdot \alpha \cdot S^*$$

$$= 2 \cdot (\ln 2 - \frac{1}{2}) \cdot p_{atm} \cdot \Delta\theta_{tot\ I} \tag{52}$$

Using $dt = \frac{1}{U_s} \cdot d\delta$ like in (24) the same mode of procedure gives ΔU_{Ia} :

$$\varrho_v \cdot S^* \cdot \Delta U_{Ia} = 2(\ln 2 - \tfrac{1}{2}) \cdot p_{atm} \cdot \frac{1}{U_s} \cdot \Delta\theta_{tot\ I}$$

$$\Delta U_{Ia} = (\ln 2 - \tfrac{1}{2}) \cdot \frac{1}{\varrho_v} \cdot p_{atm} \cdot \frac{1}{U_s} \cdot \alpha \qquad (53)$$

In the graph (fig.3) there is again (like at (24)) used a mean value of U instead of U_s.

Because the airbubbles operationally can be treated like a reduced amount of air at the waterfront acting the isothermal way like the air in front of the water in case II, the effect from the bubbles is simply included in formula (38) by putting β to:

$$\beta = 1 + \frac{\Delta\theta_{tot\ I}}{\delta_I} = 1 + \frac{\pi}{8\sqrt{2}} \cdot \sqrt{\frac{p_{atm}}{\varrho_a}} \cdot \frac{1}{U_s} \cdot \alpha \qquad (54)$$

For model size U_s there is used (33) so like in (53) (51) gives:

$$\Delta U_{ma} = 4\ (\ln 2 - \tfrac{1}{2}) \cdot \frac{1}{\varrho_v} \cdot \frac{1}{U_s} \cdot \frac{p^{*2}_{max}}{p^*_{max} + p_{atm}} \cdot \alpha \qquad (55)$$

Like in (26) and (44) a vertical velocity is created. Its effect is as in (28) and (45) which, as expected, are of the same size except for the minor difference induced by taking mean values at different states of the evaluation. Using $\frac{1}{3}$ as in (45) (53) and (55) gives the velocity reducing sizes ΔU_{Iab} and ΔU_{mab} to be used in (29) and (46):

$$\Delta U_{Iab} = \frac{4}{3} \cdot (\ln 2 - \tfrac{1}{2}) \cdot \frac{1}{\varrho_v} \cdot p_{atm} \cdot \frac{1}{U_s} \cdot \alpha \qquad (56)$$

$$\Delta U_{mab} = \frac{16}{3} \cdot (\ln 2 - \frac{1}{2}) \cdot \frac{1}{\varrho_v} \cdot \frac{1}{U_s} \cdot \frac{p^{*2}_{max}}{p^*_{max} + p_{atm}} \cdot \alpha \tag{57}$$

If there are bubbles only by the waterfront the sum of them is regarded as $\alpha \cdot S^*$ in (51). Any roughness of the waterfront is treated in the same way.

When air gets into the water its density, ϱ_v, becomes lower.

EXPLANATION TO THE GRAPH, FIG.3

In the graph U is the horizontal velocity in metres per seconds of the oncoming waterfront, and P is the maximum shock pressure in megaponds per square metres. α refers to airbubbles as mentioned by formula (51).

THE CUBIC OR AXISYMMETRIC CASE

The formulas for the axisymmetric case are deduced in the same manner as for the plane case. Now R is the radius of a cylinder of water. The main formulas are changed to the following for water without airbubbles:

(11) will be:

$$\delta_I = \frac{\sqrt{3}}{4} \cdot \sqrt{\frac{\varrho_a}{p_{atm}}} \cdot U_s \cdot R = 1.55 \cdot 10^{-3} \cdot U_s \cdot R \text{ (m)} \tag{58}$$

(23) will be:

$$S^* = \frac{2}{3\sqrt{6}} \cdot R = 0.27 \cdot R \tag{59}$$

(29) will be:

$$U_I = U_s - \frac{9\sqrt{2}}{8} \cdot \frac{1}{\varrho_v} \cdot \sqrt{\varrho_a \cdot p_{atm}} - \frac{\sqrt{2}}{3} \cdot \frac{1}{\varrho_v} \cdot \sqrt{\varrho_a \cdot p_{atm}}$$

$$= U_s - 0.75 \quad (m/s) \tag{60}$$

As this is a smaller reduction in velocity than in the plane case it is understood that the model shock pressure here is greater.

(38) will be:

$$U_I = (1 + \frac{16}{27}) \cdot 3\sqrt{6} \cdot \frac{1}{\varrho_v} \cdot p_{atm} \cdot \delta_I \cdot \frac{1}{R} \cdot \frac{1}{U_I} \cdot \left[2\ln \frac{\delta_I}{\delta_{min}} + \frac{\delta_{min}}{\delta_I} - 1\right] = 1.83 \cdot \frac{U_s}{U_I} \cdot \left[2\ln \frac{\delta_I}{\delta_{min}} + \frac{\delta_{min}}{\delta_I} - 1\right] \tag{61}$$

For the prototype case this formula tends to give a smaller pressure than in the plane case.

CONCLUSION

Although the formulas are evaluated for the idealized case in fig.1 right, the final formulas (38) and (48) reveal that the maximum pressure depends only on the velocity of the oncoming waterfront, U_s , and not on its size, R. As the parallel waterfront seems to be the most dangerous case for the plane wall, what is found here is the highest possible pressure. It will then be reached when a wall is hit by a mass of water of any shape, for instance just the top of a wave, or even only a drop of water. Then R is the size of the water from the line of symmetry in pressure.

But the described pressure will usually only affect an infinitesimal area, with much less pressures around, making it so difficult to decide a design pressure, also because the involved amount of airbubbles often is unknown.

The wall hit by the water does not need to be vertical for the use of the formulas.

NOTATION

k_i	$(= p_{atm} \cdot \delta_I)$, a constant in isothermal compression
ln	natural logarithme
$p(x_1)$	pressure above p_{atm} in the water at x_1
$p(z)$	pressure above p_{atm} in the waterfront at z
p_{abs}	absolute pressure (including p_{atm})
$p_{abs\ a}$	p_{abs} after adiabatic compression
$p_{abs\ i}$	p_{abs} after isothermal compression
$p_{abs\ u}$	p_{abs} after the outflow formula
p_{atm}	atmospheric pressure
p^*	mean value of $p(z)$ over $z = 0$ to $z = R$ (time dependent)
p^*_{max}	maximum value of p^* (the shock pressure)
t	time
u	horizontal particle velocity
w	vertical particle velocity
wf	(as index) stating that the size is at the waterfront

x	horizontal co-ordinate
x_1	horizontal co-ordinate
z	vertical co-ordinate
E_k	kinetic energy from U_I
E_p	work carried out by the water in case II
$G_x(x)$	horizontal particle acceleration at x
$G_z(z)$	vertical particle acceleration at z
R	the height of the oncoming water from a line of symmetry in pressure or the radius of an axisymmetric mass of water
S	hydrodynamic mass-length (depending on z)
S_{max}	maximum value of S (for $z = 0$)
S_1	the length over which $G_x(x)$ operationally is said to drop to 0
S^*	mean value of S over $z = 0$ to $z = R$
U	horizontal velocity of the hydrodynamic mass (time dependent)
U_s	U at great distance from the wall (start velocity)
U_I	horizontal velocity of the waterfront at $\delta = \delta_I$
V_I	generated vertical velocity when $\delta = \delta_I$ (z dependent)
$V_{I\ top}$	V_I at the free water surface ($z = R$)
$V_{m\ top}$	generated maximum vertical velocity in the model case
V_2	generated vertical velocity in case II (z dependent)
$V_{2\ top}$	V_2 at the free water surface ($z = R$)
$W_a(z)$	vertical velocity of the air particle in the slit at z
α	relative content of airbubbles in the water
β	air-regarding coefficient in case II

δ distance between waterfront and wall (time dependent)

δ_{min} δ at maximum pressure

δ_I δ at $p^* = p_{atm}$ $(p_{abs} = 2p_{atm})$

$\theta_p(x)$ volume of airbubbles per unit cube at x at pressure = $p(x_1)$

$\theta_o(x)$ volume of airbubbles per unit cube at x at pressure = 0

π = 3.14159

ϱ_a density of the air at pressure = p_{atm}

ϱ_v density of the water

ΔI change in horizontal momentum

$\Delta I(z)$ ΔI at z

ΔU_{bm} horizontal influence from generated vertical velocity in model case

ΔU_{b1} horizontal influence from generated vertical velocity in case I

ΔU_{b2} horizontal influence from generated vertical velocity in case II

ΔU_m reduction in horizontal velocity of hydrodynamic mass in model case

ΔU_{ma} ΔU_m from airbubbles or rough waterfront

ΔU_{mab} total reduction in horizontal velocity of the waterfront because of airbubbles in model case

ΔU_I reduction in horizontal velocity of the hydrodynamic mass in case I

ΔU_{Ia} ΔU_I from airbubbles or rough waterfront

ΔU_{Iab} total reduction in horizontal velocity of the waterfront because of airbubbles in case I

ΔU_2	reduction of horizontal velocity of the hydrodynamic mass in case II
$\Delta\theta(x)$	reduction in airbubble volume per unit cube at x
$\Delta\theta_{tot}$	total reduction in airbubble volume per square unit of the waterfront (time dependent)
$\Delta\theta_{totI}$	$\Delta\theta_{tot}$ for $\delta = \delta_I$
$\Delta\tau$	duration of the shock