CHAPTER X

FORMULAS AND TABLES
FOR THE PROGRESSIVE CNOIDAL WAVE ON ARBITRARY DEPTH

ABSTRACTS

In this chapter the final formulas for the cnoidal waves on arbitrary depth of chapter IX is given. The most important formulas are tabulated. A numerical example shows the use of the table and the formulas.

INTRODUCTION

Some of the formulas found in chapter IX may be a little difficult to use in practice, because they involve elliptic functions. So they are tabulated. This is of special importance for the energy transport, which is needed for the calculation of shoaling.

For the engineer it may not be of so big interest to know how to develope a wave theory and how to find the formulas. It is mainly of interest that the formulas given are correct. In an appendix we show that eqs. D1 - D19 for the deep water cnoidal wave are correct second order expressions.

Eqs. F1 - F2o describe the general situation. Eqs. D4 - D19 give the same formulas for the more simple case with infinite deep water. The other extreme of the general situation is the solitary wave, given in eqs. E1 - E17. For small amplitude waves the cnoidal wave in eqs. F1 - F2o will approximate the sinusoidal wave in eqs. S1 - S17.

Further the formulas are given as non-dimensional quantities and tabulated.

Tables to assist in numerical calculations with regular waves have been in widely use for many years. For the waves here it may be even more desirable with tables, because elliptic functions are involved. But with the widespread use of handy computers the need for tables is decreasing, though.

The celerity c as given in eq. F7 is a correct second order expression improved by a third order term which is only determined by the conditions at the surface of the second order wave. This third order term is affected by second order rotation, as explained in chapter IX. For the table we have selected the situation with irrotational waves. This makes the celerity of eq. T1o a little different from eq. F1o, a difference of a few percent and of third order magnitude. When this celerity becomes smaller than the sinusoidal celerity, we have chosen the sinusoidal celerity. This is believed to be more correct.

The deep water sinusoidal expressions for the celerity and wave length, $c_o = L_o/T$ and $L_o = T^2 g/2\pi$, are used as reference quantities in the table, which is felt practical for waves with a given period T.

In the formulas for deep water waves, eqs. D4 - D19, the vertical coordinate z, and the actual water depth y will be infinite, but this gives no problems as only y - z, the depth below the surface, is used.

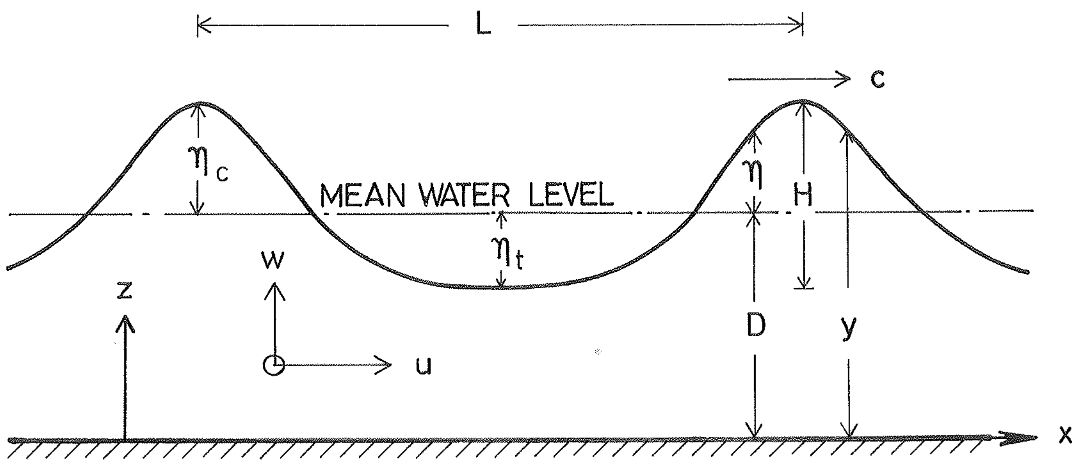

Fig. 1. Definition sketch.

NOTATION

c	$= L/T$, the wave celerity.
c_o	$= L_o/T$, the wave celerity for the sinusoidal deep water wave.
cn	= Jacobian elliptic cosine function.
cosh	= hyperbolic cosine function.
coth	= 1/tanh = cosh/sinh
g	$= 9{,}8$ meters/second^2, acceleration of gravity.
k	$= 2\pi/L$
m	= parameter of elliptic functions (= square of the modulus).
p	$= p(x,z,t)$, the water pressure at a point, excluding atmospheric pressure from the air above the water.
r	= see eqs. F7, E7, and S7.
sinh	= hyperbolic sine function.
t	= the time.
tanh	= sinh/cosh.
u	$= u(x,z,t)$, horizontal particle velocity.
u_{rot}	= see eqs F19 and D19, horizontal particle velocity in a wave with rotation of arbitrary second order magnitude.
w	$= w(x,z,t)$, vertical particle velocity.
x	= horizontal coordinate.
y	$= D + \eta$, actual water depth.
z	= vertical coordinate
D	= mean water depth.
E	= complete elliptic integral of the second kind.
E_{flux}	= energy flux through a vertical with the mean water as reference level
H	= the wave height.
K	= complete elliptic integral of the first kind.
L	= the wave length.
L_o	$= \frac{g}{2\pi} T^2$, the deep water sinusoidal wave length.

R	=	see eqs. F8, D8, E8, and S8.
R_c	=	see eq. F9, R at the crest of the wave.
T	=	the wave period.
β	=	see eqs. F5, D5, and E5.
γ	=	ϱg, unit weight of water.
δ	=	see eqs. F19 and D19, arbitrary constant, not much bigger than 1, to obtain second order magnitude rotation. For $\delta = \frac{1}{2}$ the wave is irrotational.
η	=	$y - D$, surface elevation.
η_c	=	see eqs. F3, E3, and S3, the crest height above mean water level.
η_t	=	see eqs. F2, E2, and S2, the negative trough depth.
θ	=	see eqs. F17 and E17.
π	=	3.14159
ϱ	=	γ/g, unit mass of the water.
Δu	=	see eqs. F2o and F19.
Δu_s	=	see eq. T2os, and T1o, Δu at the surface $z = y$.

FORMULAS FOR THE CNOIDAL WAVE ON ARBITRARY DEPTH

$$\eta = H\,\mathrm{cn}^2 \frac{2K}{L}(x - ct) + \eta_t \tag{F1}$$

$$\eta_t = \eta_c - H \tag{F2}$$

$$\eta_c = \frac{H}{m}\left(1 - \frac{E}{K}\right) \tag{F3}$$

$$mK^2 = \pi^3 \frac{H}{L}\left[\coth kD + \frac{2}{3}\beta\right] = \pi^3 \frac{H}{L}\coth^3 kD \tag{F4}$$

$$\beta = \frac{3}{2}\,\frac{\coth kD}{\sinh^2 kD} \tag{F5}$$

$$k = \frac{2\pi}{L} \tag{F6}$$

$$r = \frac{4K}{L}\sqrt{1 - \frac{m}{2}} \tag{F7}$$

$$R = r\left(1 + \frac{1}{5}\tanh 5\beta \frac{k^3}{r^2}\eta\right) \tag{F8}$$

$$R_c = R \quad \text{for} \quad \eta = \eta_c \tag{F9}$$

$$c = \sqrt{\frac{g}{R_c}\tanh R_c(D + \eta_c) + g\left(\eta_c - \frac{H}{2}\right)} \tag{F10}$$

$$y = D + \eta \tag{F11}$$

$$u = c\eta R \frac{\cosh Rz}{\sinh Ry} \tag{F12}$$

$$w = c\frac{\partial \eta}{\partial x}\left[-1+\eta R \coth Ry\right]\frac{\sinh Rz}{\sinh Ry} \tag{F13}$$

$$\frac{p}{\gamma} = y - z + \frac{c^2}{g}\left\{\left[\frac{\partial^2\eta}{\partial x^2}\frac{1}{R} - \left[2\left(\frac{\partial\eta}{\partial x}\right)^2 + \eta\frac{\partial^2\eta}{\partial x^2}\right]\coth Ry\right]\cdot \frac{\cosh Ry - \cosh Rz}{\sinh Ry} + \frac{1}{4}\left[\left(\frac{\partial\eta}{\partial x}\right)^2 - \eta\frac{\partial^2\eta}{\partial x^2}\right]\frac{\cosh 2Ry - \cosh 2Rz}{\sinh^2 Ry}\right\} \tag{F14}$$

$$\frac{\partial\eta}{\partial x} = -4\frac{H}{L}K\sqrt{cn^2\theta(1-cn^2\theta)(1-m+m\,cn^2\theta)} \tag{F15}$$

$$\frac{\partial^2\eta}{\partial x^2} = -8\frac{H}{L^2}K^2\left[m-1-2(2m-1)cn^2\theta+3m\,cn^4\theta\right] \tag{F16}$$

$$\theta = \frac{2K}{L}(x-ct) \tag{F17}$$

$$E_{flux} = \gamma c\eta^2 + \rho c^3\eta\left\{\frac{\partial^2\eta}{\partial x^2}\frac{1}{2R}\left[\coth Ry - \frac{Ry}{\sinh^2 Ry}\right] + \left[\left(\frac{\partial\eta}{\partial x}\right)^2 + \frac{1}{2}\eta\frac{\partial^2\eta}{\partial x^2}\right]\frac{Ry\coth Ry}{\sinh^2 Ry} + \left(\frac{\partial\eta}{\partial x}\right)^2\left[\frac{1}{2} - \coth^2 Ry\right] - \eta\frac{\partial^2\eta}{\partial x^2}\coth^2 Ry\right\} \tag{F18}$$

$$u_{rot} = c\eta R\frac{\cosh Rz}{\sinh Ry} + \Delta u\cdot\delta \tag{F19}$$

$$\Delta u = c\left(\frac{H}{2}\right)^2 R^2 \coth RD\left[\frac{\cosh Rz}{\sinh Ry} - \frac{1}{Ry}\right] \tag{F20}$$

DEEP WATER LIMIT

$D \to \infty$; $D/L \to \infty$; $kD \to \infty$

Eqs. F1, F2, F3, F6, F7, F11, F15, F16, F17 are unchanged.
The others will be

$$mK^2 = \pi^3 \frac{H}{L} \tag{D4}$$

$$\beta = 0 \tag{D5}$$

$$R = r \tag{D8}$$

$$c = \sqrt{\frac{g}{r} + g\left(\eta_c - \frac{H}{2}\right)} \tag{D10}$$

$$u = c\eta r\, e^{r(z-y)} \tag{D12}$$

$$w = c\frac{\partial\eta}{\partial x}\left[-1+\eta r\right]e^{r(z-y)} \tag{D13}$$

$$\frac{p}{\gamma} = y - z + \frac{c^2}{g}\left\{\left[\frac{\partial^2\eta}{\partial x^2}\frac{1}{r} - \left[2\left(\frac{\partial\eta}{\partial x}\right)^2 + \eta\frac{\partial^2\eta}{\partial x^2}\right]\right]\left[1 - e^{r(z-y)}\right] + \frac{1}{2}\left[\left(\frac{\partial\eta}{\partial x}\right)^2 - \eta\frac{\partial^2\eta}{\partial x^2}\right]\left[1 - e^{2r(z-y)}\right]\right\} \tag{D14}$$

$$\frac{p}{\gamma} = D - z + \eta\, e^{k(z-y)} + \frac{\pi}{4} H \frac{H}{L}\left[e^{k(z-y)} - e^{2k(z-y)}\right] \tag{D14a}$$

$$E_{flux} = \gamma c\eta^2 + \rho c^3\eta\left\{\frac{\partial^2\eta}{\partial x^2}\frac{1}{2r} - \frac{1}{2}\left(\frac{\partial\eta}{\partial x}\right)^2 - \eta\frac{\partial^2\eta}{\partial x^2}\right\} \tag{D18}$$

$$u_{rot} = c\eta r e^{r(z-y)} + \delta\, c \left(\frac{H}{2}\right)^2 r^2 e^{r(z-y)} \tag{D19}$$

SOLITARY WAVE LIMIT

$L/D \rightarrow \infty$

Eqs. F9, F11, F12, F13, F19, F2o are unchanged. In eq. F14 for the pressure the last term (with cosh 2Rz) should be neglected for $H/D \gtrsim 0.5$. The other equations will be

$$\eta = H \operatorname{sech}^2 \sqrt{\frac{H}{D}} \frac{\sqrt{2}}{2D}(x-ct) = \frac{H}{\cosh^2\theta} \tag{E1}$$

$$\eta_t = 0 \tag{E2}$$

$$\eta_c = H \tag{E3}$$

$$m \rightarrow 1 \quad ; K \rightarrow \infty \; ; mK^2 \rightarrow \frac{1}{8}\frac{H}{D}\left(\frac{L}{D}\right)^2 \tag{E4}$$

$$\beta \rightarrow \infty \; ; \quad \beta \rightarrow \frac{3}{2}\frac{1}{(kD)^3} \tag{E5}$$

$$k \rightarrow 0 \tag{E6}$$

$$r = \frac{1}{D}\sqrt{\frac{H}{D}} \tag{E7}$$

$$R = r\left(1 + \frac{1}{5}\tanh\frac{15}{2}\frac{\eta}{H}\right) \tag{E8}$$

$$c = \sqrt{\frac{g}{R_c}\tanh R_c(D+H) + g\frac{H}{2}} \tag{E1o}$$

$$\frac{\partial\eta}{\partial x} = -\sqrt{2}\frac{H}{D}\sqrt{\frac{H}{D}}\;\frac{\sinh\theta}{\cosh^3\theta} = -\sqrt{2}\;\frac{\eta}{D}\sqrt{\frac{H}{D}-\frac{\eta}{D}} \tag{E15}$$

$$\frac{\partial^2\eta}{\partial x^2} = -\;\frac{H^2}{D^3}\left(\frac{3}{\cosh^4\theta} - \frac{2}{\cosh^2\theta}\right) = \frac{1}{D}\frac{\eta}{D}\left(2\frac{H}{D} - 3\frac{\eta}{D}\right) \tag{E16}$$

$$\theta = \sqrt{\frac{H}{D}}\;\frac{\sqrt{2}}{2D}(x-ct) \tag{E17}$$

SINUSOIDAL WAVE LIMIT

Both $\frac{H}{L} \rightarrow o$ and $\frac{H}{D}(\frac{L}{D})^2 \rightarrow o$ gives $m \rightarrow o$, see eqs. F4, D4 and E4. For $m = o$ we have $K = \frac{\pi}{2}$ and $E = \frac{\pi}{2}$, and cn = cos. Eqs. F5, F6, F11 are unchanged, but eqs. F18, F19, F2o contain second order terms that need not to be included in the considerations here.

$$\eta = H\cos^2\frac{\pi}{L}(x-ct) - \frac{H}{2} = \frac{H}{2}\cos\frac{2\pi}{L}(x-ct) \tag{S1}$$

$$\eta_t = -\frac{H}{2} \tag{S2}$$

$$\eta_c = \frac{H}{2} \tag{S3}$$

$$mK^2 \rightarrow 0 \qquad \text{for } \frac{H}{L} \rightarrow 0 \ \wedge \ \frac{H}{D}\left(\frac{L}{D}\right)^2 \rightarrow 0 \tag{S4}$$

$$r = \frac{2\pi}{L} = k \tag{S7}$$

$$R = r \tag{S8}$$

$$c = \sqrt{\frac{g}{k}\tanh kD} \tag{S1o}$$

$$u = c\eta R\,\frac{\cosh Rz}{\sinh Ry} \tag{S12}$$

$$w = -c\frac{\partial\eta}{\partial x}\,\frac{\sinh Rz}{\sinh Ry} \tag{S13}$$

$$\frac{p}{\gamma} = y - z + \frac{c^2}{gR}\frac{\partial^2\eta}{\partial x^2}\,\frac{\cosh Ry - \cosh Rz}{\sinh Ry}$$

$$= D - z + \eta\left[1 - \tanh kD\,\frac{\cosh ky - \cosh kz}{\sinh ky}\right] \tag{S14}$$

$$\frac{\partial\eta}{\partial x} = -\frac{H}{2}k\sin k(x-ct) \tag{S15}$$

$$\frac{\partial^2\eta}{\partial x^2} = -\frac{H}{2}k^2\cos k(x-ct) = -\eta k^2 \tag{S16}$$

$$\theta = \frac{\pi}{L}(x - ct) \tag{S17}$$

TABLE FORMULAS

We will here prepare some of the formulas for the table. The nondimensional quantities will be given as a function of the wave steepness H/L and of D/L. It should be noted that the celerity used in eq. T1o is for irrotational waves as found in appendix IV of chapter IX, while the third order influence of rotation was neglected in eq. F1o. Eqs. F4, F5, will not be rewritten. The others will in non-dimensional form be given as

$$\frac{\eta}{H} = cn^2\theta + \frac{\eta_t}{H} \tag{T1}$$

$$\frac{\eta_t}{H} = \frac{\eta_c}{H} - 1 \tag{T2}$$

$$\frac{\eta_c}{H} = \frac{1}{m}\left(1 - \frac{E}{K}\right) \tag{T3}$$

$$kL = 2\pi \tag{T6}$$

$$rL = 4K\sqrt{1 - \frac{m}{2}} \tag{T7}$$

$$RL = rL\left(1 + \frac{1}{5}\tanh 5\beta \frac{(2\pi)^3}{(rL)^2}\frac{H}{L}\frac{\eta}{H}\right) \tag{T8}$$

$$\frac{c}{c_o} = \left[1 - \frac{\Delta u_s}{c}\right]\frac{2\pi}{R_cL}\tanh R_cL\left(\frac{D}{L} + \frac{H}{L}\frac{\eta_c}{H}\right) + 2\pi\frac{H}{L}\left(\frac{\eta_c}{H} - \frac{1}{2}\right) \tag{T1o}$$

If this equation gives a value less than the sinusoidal, the sinusoidal value is chosen for the table.

$$\frac{\Delta u_s}{c} = \frac{1}{8}\left(\frac{H}{L}\right)^2 rL\coth rL\frac{D}{L}\left[rL\coth rL\frac{D}{L} - \frac{L}{D}\right] \tag{T2os}$$

$$\frac{y}{L} = \frac{D}{L} + \frac{H}{L}\frac{\eta}{H} \tag{T11}$$

Eqs. F12, F13, F14 can be changed in the same way, when wanted. Eq. F16 is made dimensionless by considering $\frac{\partial^2\eta}{\partial x^2}$ L. With $c^2/(g\eta)$ determined from eq. F1o, eq. F18 will be

$$\frac{E_{flux}}{\gamma c_0 L_0^2} = \frac{c}{c_0}\left(\frac{H}{L_0}\right)^2\left(\frac{\eta}{H}\right)^2\left\{1+\frac{\tanh rL\left(\frac{D}{L}+\frac{H}{L}\frac{\eta_c}{H}\right)+rL\frac{H}{L}\left(\frac{\eta_c}{H}-\frac{1}{2}\right)}{rL\frac{H}{L}\frac{\eta}{H}}\cdot\right.$$

$$\left\{\frac{\partial^2\eta}{\partial x^2}L\frac{1}{2RL}\left[\coth RL\frac{y}{L}-\frac{RL\frac{y}{L}}{\sinh^2 RL\frac{y}{L}}\right]+\left[\left(\frac{\partial\eta}{\partial x}\right)^2+\frac{1}{2}\frac{H}{L}\frac{\eta}{H}\frac{\partial^2\eta}{\partial x^2}L\right]\cdot\right.$$

$$\left.\left.\frac{RL\frac{y}{L}\coth RL\frac{y}{L}}{\sinh^2 RL\frac{y}{L}}+\left(\frac{\partial\eta}{\partial x}\right)^2\left[\frac{1}{2}-\coth^2 RL\frac{y}{L}\right]-\frac{H}{L}\frac{\eta}{H}\frac{\partial^2\eta}{\partial x^2}L\coth^2 RL\frac{y}{L}\right\}\right\} \quad \text{(T18)}$$

where $\frac{H}{L_0} = \frac{H}{L}\frac{L}{L_0} = \frac{H}{L}\frac{c}{c_0}$

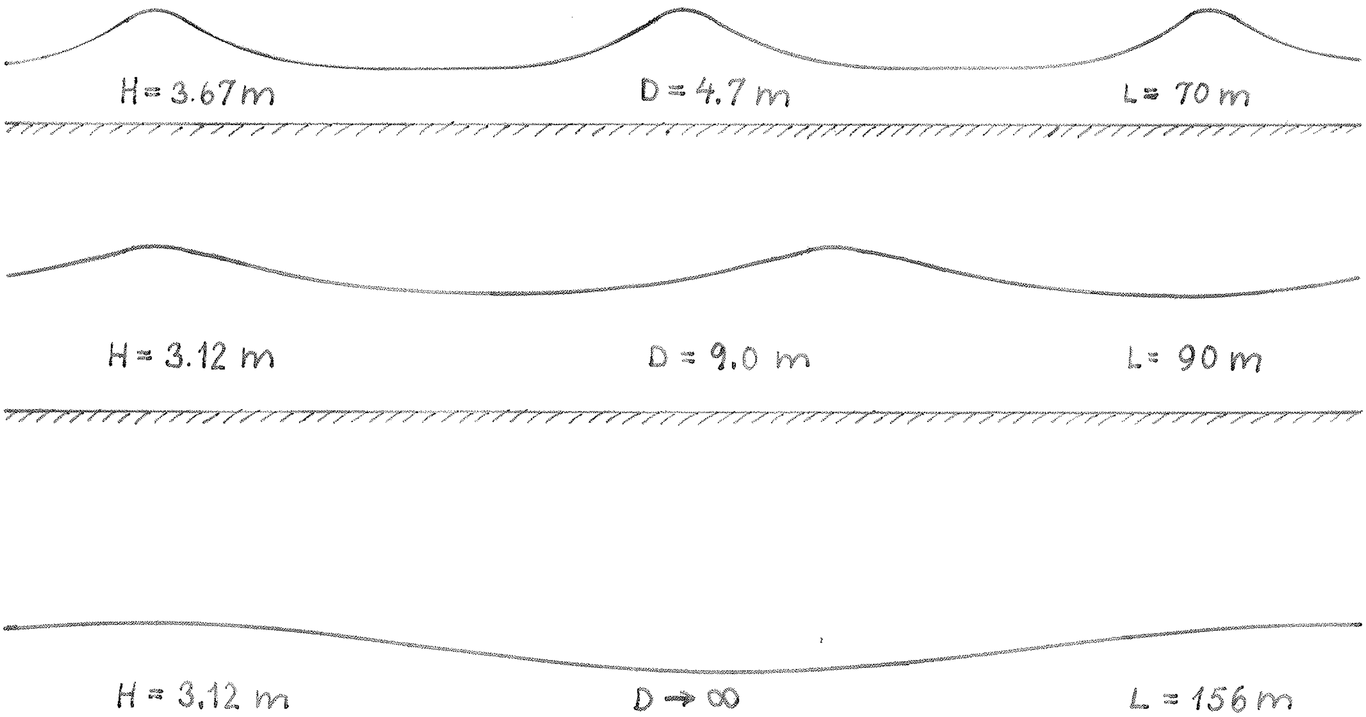

Fig. 2. The wave with the period T = 1o seconds, considered in appendix I, propagating towards the coast. The vertical scale is twice as big as the horizontal scale.

TABLE for PROGRESSIVE CNOIDAL WAVES

1%

H/L_o = 1%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.012	0.0399	0.0332	0.30	0.88	23.63	95.52	0.232E-05	0.833	8.35	1.00
0.013	0.0418	0.0321	0.31	0.87	21.71	82.84	0.246E-05	0.769	7.68	1.00
0.014	0.0437	0.0312	0.32	0.86	20.08	72.59	0.261E-05	0.714	7.10	1.00
0.015	0.0455	0.0303	0.33	0.85	18.69	64.29	0.276E-05	0.667	6.61	1.00
0.016	0.0472	0.0295	0.34	0.84	17.48	57.44	0.291E-05	0.625	6.18	1.00
0.017	0.0489	0.0288	0.35	0.83	16.41	51.55	0.307E-05	0.588	5.80	1.00
0.018	0.0506	0.0281	0.36	0.82	15.48	46.68	0.322E-05	0.556	5.47	1.00
0.019	0.0521	0.0274	0.36	0.81	14.67	42.63	0.336E-05	0.526	5.19	1.00
0.020	0.0538	0.0269	0.37	0.80	13.92	38.88	0.351E-05	0.500	4.92	1.00
0.025	0.0612	0.0245	0.41	0.75	11.17	26.35	0.417E-05	0.400	3.94	0.99
0.030	0.0681	0.0227	0.44	0.70	9.46	19.08	0.472E-05	0.333	3.31	0.98
0.035	0.0747	0.0213	0.47	0.67	8.37	14.46	0.515E-05	0.286	2.89	0.95
0.040	0.0810	0.0202	0.49	0.64	7.69	11.34	0.548E-05	0.250	2.60	0.90
0.045	0.0870	0.0193	0.52	0.61	7.25	9.13	0.575E-05	0.222	2.39	0.85
0.050	0.0928	0.0186	0.54	0.60	6.96	7.51	0.598E-05	0.200	2.24	0.79
0.060	0.1038	0.0173	0.58	0.57	6.65	5.35	0.634E-05	0.167	2.05	0.68
0.070	0.1139	0.0163	0.61	0.55	6.50	4.03	0.663E-05	0.143	1.93	0.58
0.080	0.1232	0.0154	0.65	0.54	6.43	3.17	0.688E-05	0.125	1.86	0.51
0.090	0.1322	0.0147	0.68	0.54	6.38	2.55	0.707E-05	0.111	1.81	0.44
0.100	0.1410	0.0141	0.71	0.53	6.35	2.09	0.723E-05	0.100	1.77	0.39
0.110	0.1496	0.0136	0.74	0.53	6.34	1.73	0.734E-05	0.091	1.74	0.35
0.120	0.1581	0.0132	0.76	0.52	6.33	1.46	0.743E-05	0.083	1.72	0.32
0.130	0.1666	0.0128	0.78	0.52	6.32	1.23	0.749E-05	0.077	1.71	0.29
0.140	0.1750	0.0125	0.80	0.52	6.31	1.05	0.752E-05	0.071	1.69	0.26
0.150	0.1833	0.0122	0.82	0.52	6.31	0.90	0.754E-05	0.067	1.68	0.24
0.160	0.1916	0.0120	0.83	0.52	6.30	0.78	0.755E-05	0.063	1.67	0.23
0.170	0.2000	0.0118	0.85	0.52	6.30	0.68	0.754E-05	0.059	1.67	0.21
0.180	0.2083	0.0116	0.86	0.51	6.30	0.59	0.752E-05	0.056	1.66	0.20
0.190	0.2167	0.0114	0.88	0.51	6.30	0.51	0.750E-05	0.053	1.66	0.19
0.200	0.2251	0.0113	0.89	0.51	6.30	0.45	0.746E-05	0.050	1.65	0.18
0.220	0.2421	0.0110	0.91	0.51	6.29	0.35	0.738E-05	0.045	1.64	0.17
0.240	0.2592	0.0108	0.93	0.51	6.29	0.27	0.728E-05	0.042	1.64	0.16
0.260	0.2766	0.0106	0.94	0.51	6.29	0.21	0.718E-05	0.038	1.63	0.15
0.280	0.2942	0.0105	0.95	0.51	6.29	0.16	0.707E-05	0.036	1.63	0.14
0.300	0.3121	0.0104	0.96	0.51	6.29	0.13	0.697E-05	0.033	1.63	0.14
0.320	0.3303	0.0103	0.97	0.51	6.29	0.10	0.688E-05	0.031	1.63	0.13
0.340	0.3486	0.0103	0.98	0.51	6.29	0.08	0.679E-05	0.029	1.63	0.13
0.360	0.3672	0.0102	0.98	0.51	6.29	0.06	0.671E-05	0.028	1.62	0.13
0.380	0.3860	0.0102	0.98	0.51	6.29	0.05	0.664E-05	0.026	1.62	0.13
0.400	0.4050	0.0101	0.99	0.51	6.29	0.04	0.658E-05	0.025	1.62	0.12
0.420	0.4241	0.0101	0.99	0.51	6.29	0.03	0.653E-05	0.024	1.62	0.12
0.450	0.4530	0.0101	0.99	0.51	6.29	0.02	0.646E-05	0.022	1.62	0.12
0.500	0.5018	0.0100	1.00	0.51	6.29	0.01	0.639E-05	0.020	1.62	0.12
0.600	0.6006	0.0100	1.00	0.51	6.29	0.00	0.631E-05	0.017	1.62	0.12
0.700	0.7002	0.0100	1.00	0.51	6.29	0.00	0.628E-05	0.014	1.62	0.12
1.000	1.0000	0.0100	1.00	0.51	6.29	0.00	0.627E-05	0.010	1.62	0.12
INFINIT	INFINITE	0.0100	1.00	0.51	6.29	0	0.627E-05	0	1.62	0.12

L_o and c_o are first order sinusoidal deep water values (L_o = 1.56 T^2 metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

2%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 2%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.025	0.0595	0.0476	0.42	0.82	16.09	28.67	0.164E-04	0.800	5.69	1.00
0.030	0.0666	0.0444	0.45	0.79	13.36	20.46	0.183E-04	0.667	4.72	1.00
0.035	0.0730	0.0417	0.48	0.76	11.50	15.51	0.200E-04	0.571	4.06	1.00
0.040	0.0791	0.0396	0.51	0.72	10.17	12.16	0.215E-04	0.500	3.57	0.99
0.045	0.0850	0.0378	0.53	0.70	9.21	9.79	0.228E-04	0.444	3.21	0.97
0.050	0.0908	0.0363	0.55	0.67	8.50	8.03	0.239E-04	0.400	2.94	0.95
0.060	0.1020	0.0340	0.59	0.63	7.59	5.64	0.256E-04	0.333	2.55	0.89
0.070	0.1127	0.0322	0.62	0.60	7.10	4.16	0.267E-04	0.286	2.31	0.82
0.080	0.1228	0.0307	0.65	0.58	6.82	3.20	0.276E-04	0.250	2.16	0.75
0.090	0.1322	0.0294	0.68	0.57	6.66	2.55	0.284E-04	0.222	2.05	0.68
0.100	0.1410	0.0282	0.71	0.56	6.56	2.09	0.291E-04	0.200	1.98	0.63
0.110	0.1496	0.0272	0.74	0.55	6.49	1.73	0.296E-04	0.182	1.92	0.57
0.120	0.1581	0.0264	0.76	0.55	6.45	1.46	0.300E-04	0.167	1.88	0.53
0.130	0.1666	0.0256	0.78	0.54	6.42	1.23	0.302E-04	0.154	1.85	0.49
0.140	0.1750	0.0250	0.80	0.54	6.39	1.05	0.304E-04	0.143	1.82	0.46
0.150	0.1833	0.0244	0.82	0.53	6.37	0.90	0.305E-04	0.133	1.80	0.43
0.160	0.1916	0.0240	0.83	0.53	6.36	0.78	0.305E-04	0.125	1.78	0.40
0.170	0.2000	0.0235	0.85	0.53	6.35	0.68	0.305E-04	0.118	1.76	0.38
0.180	0.2083	0.0231	0.86	0.53	6.34	0.59	0.304E-04	0.111	1.75	0.36
0.190	0.2167	0.0228	0.88	0.53	6.34	0.51	0.303E-04	0.105	1.74	0.35
0.200	0.2251	0.0225	0.89	0.53	6.33	0.45	0.302E-04	0.100	1.73	0.33
0.220	0.2421	0.0220	0.91	0.52	6.32	0.35	0.298E-04	0.091	1.72	0.31
0.240	0.2592	0.0216	0.93	0.52	6.32	0.27	0.294E-04	0.083	1.71	0.29
0.260	0.2766	0.0213	0.94	0.52	6.31	0.21	0.290E-04	0.077	1.70	0.27
0.280	0.2942	0.0210	0.95	0.52	6.31	0.16	0.286E-04	0.071	1.69	0.26
0.300	0.3121	0.0208	0.96	0.52	6.31	0.13	0.281E-04	0.067	1.69	0.25
0.320	0.3303	0.0206	0.97	0.52	6.31	0.10	0.277E-04	0.063	1.68	0.25
0.340	0.3486	0.0205	0.98	0.52	6.31	0.08	0.274E-04	0.059	1.68	0.24
0.360	0.3672	0.0204	0.98	0.52	6.30	0.06	0.271E-04	0.056	1.68	0.24
0.380	0.3860	0.0203	0.98	0.52	6.30	0.05	0.268E-04	0.053	1.68	0.23
0.400	0.4050	0.0202	0.99	0.52	6.30	0.04	0.265E-04	0.050	1.68	0.23
0.420	0.4241	0.0202	0.99	0.52	6.30	0.03	0.263E-04	0.048	1.67	0.23
0.450	0.4530	0.0201	0.99	0.52	6.30	0.02	0.260E-04	0.044	1.67	0.23
0.500	0.5018	0.0201	1.00	0.52	6.30	0.01	0.257E-04	0.040	1.67	0.22
0.600	0.6006	0.0200	1.00	0.52	6.30	0.00	0.254E-04	0.033	1.67	0.22
0.700	0.7002	0.0200	1.00	0.52	6.30	0.00	0.253E-04	0.029	1.67	0.22
1.000	1.0000	0.0200	1.00	0.52	6.30	0.00	0.253E-04	0.020	1.67	0.22
INFINIT	INFINITE	0.0200	1.00	0.52	6.30	0	0.253E-04	0	1.67	0.22

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

3%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 3%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	K(m)	m
0.035	0.0715	0.0613	0.49	0.80	14.27	16.52	0.473E-04	0.857	5.04	1.00
0.040	0.0779	0.0584	0.51	0.77	12.48	12.75	0.499E-04	0.750	4.41	1.00
0.045	0.0840	0.0560	0.54	0.75	11.15	10.17	0.524E-04	0.667	3.93	0.99
0.050	0.0897	0.0538	0.56	0.72	10.14	8.32	0.547E-04	0.600	3.56	0.99
0.060	0.1008	0.0504	0.59	0.68	8.76	5.85	0.584E-04	0.500	3.04	0.96
0.070	0.1114	0.0478	0.63	0.65	7.92	4.31	0.612E-04	0.429	2.70	0.92
0.080	0.1218	0.0457	0.66	0.62	7.40	3.28	0.631E-04	0.375	2.46	0.87
0.090	0.1320	0.0440	0.68	0.60	7.08	2.56	0.646E-04	0.333	2.30	0.82
0.100	0.1410	0.0423	0.71	0.59	6.87	2.09	0.661E-04	0.300	2.19	0.77
0.110	0.1496	0.0408	0.74	0.58	6.74	1.73	0.674E-04	0.273	2.10	0.72
0.120	0.1581	0.0395	0.76	0.57	6.64	1.46	0.683E-04	0.250	2.04	0.67
0.130	0.1666	0.0384	0.78	0.56	6.57	1.23	0.690E-04	0.231	1.99	0.63
0.140	0.1750	0.0375	0.80	0.56	6.52	1.05	0.694E-04	0.214	1.95	0.60
0.150	0.1833	0.0367	0.82	0.55	6.48	0.90	0.696E-04	0.200	1.91	0.57
0.160	0.1916	0.0359	0.83	0.55	6.46	0.78	0.697E-04	0.188	1.89	0.54
0.170	0.2000	0.0353	0.85	0.54	6.43	0.68	0.696E-04	0.176	1.86	0.51
0.180	0.2083	0.0347	0.86	0.54	6.41	0.59	0.694E-04	0.167	1.85	0.49
0.190	0.2167	0.0342	0.88	0.54	6.40	0.51	0.692E-04	0.158	1.83	0.47
0.200	0.2251	0.0338	0.89	0.54	6.39	0.45	0.688E-04	0.150	1.82	0.45
0.220	0.2421	0.0330	0.91	0.53	6.37	0.35	0.680E-04	0.136	1.79	0.42
0.240	0.2592	0.0324	0.93	0.53	6.36	0.27	0.671E-04	0.125	1.78	0.40
0.260	0.2766	0.0319	0.94	0.53	6.35	0.21	0.661E-04	0.115	1.77	0.38
0.280	0.2942	0.0315	0.95	0.53	6.34	0.16	0.651E-04	0.107	1.76	0.37
0.300	0.3121	0.0312	0.96	0.53	6.34	0.13	0.641E-04	0.100	1.75	0.36
0.320	0.3303	0.0310	0.97	0.53	6.34	0.10	0.632E-04	0.094	1.74	0.35
0.340	0.3486	0.0308	0.98	0.53	6.33	0.08	0.624E-04	0.088	1.74	0.34
0.360	0.3672	0.0306	0.98	0.53	6.33	0.06	0.616E-04	0.083	1.73	0.33
0.380	0.3860	0.0305	0.98	0.52	6.33	0.05	0.610E-04	0.079	1.73	0.33
0.400	0.4050	0.0304	0.99	0.52	6.33	0.04	0.604E-04	0.075	1.73	0.33
0.420	0.4241	0.0303	0.99	0.52	6.33	0.03	0.599E-04	0.071	1.73	0.32
0.450	0.4530	0.0302	0.99	0.52	6.33	0.02	0.593E-04	0.067	1.73	0.32
0.500	0.5018	0.0301	1.00	0.52	6.33	0.01	0.586E-04	0.060	1.72	0.32
0.600	0.6000	0.0300	1.00	0.52	6.33	0.00	0.579E-04	0.050	1.72	0.31
0.700	0.6993	0.0300	1.00	0.52	6.32	0.00	0.576E-04	0.043	1.72	0.31
1.000	0.9984	0.0300	1.00	0.52	6.32	0.00	0.575E-04	0.030	1.72	0.31
INFINIT	INFINITE	0.0299	1.00	0.52	6.32	0	0.576E-04	0	1.72	0.31

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

4%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 4% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.045	0.0822	0.0731	0.55	0.78	13.02	10.83	0.997E-04	0.889	4.60	1.00
0.050	0.0884	0.0707	0.57	0.76	11.73	8.70	0.103E-03	0.800	4.14	1.00
0.060	0.0999	0.0666	0.60	0.72	9.94	6.01	0.108E-03	0.667	3.49	0.98
0.070	0.1106	0.0632	0.63	0.68	8.81	4.41	0.112E-03	0.571	3.06	0.96
0.080	0.1211	0.0605	0.66	0.65	8.07	3.34	0.115E-03	0.500	2.76	0.93
0.090	0.1313	0.0583	0.69	0.63	7.59	2.61	0.118E-03	0.444	2.55	0.89
0.100	0.1410	0.0564	0.71	0.61	7.26	2.09	0.119E-03	0.400	2.40	0.85
0.110	0.1496	0.0544	0.74	0.60	7.05	1.73	0.122E-03	0.364	2.29	0.81
0.120	0.1581	0.0527	0.76	0.59	6.89	1.46	0.123E-03	0.333	2.20	0.77
0.130	0.1666	0.0513	0.78	0.58	6.78	1.23	0.125E-03	0.308	2.13	0.74
0.140	0.1750	0.0500	0.80	0.57	6.70	1.05	0.126E-03	0.286	2.08	0.70
0.150	0.1833	0.0489	0.82	0.57	6.63	0.90	0.126E-03	0.267	2.03	0.67
0.160	0.1916	0.0479	0.83	0.56	6.58	0.78	0.126E-03	0.250	2.00	0.64
0.170	0.2000	0.0471	0.85	0.56	6.54	0.68	0.126E-03	0.235	1.97	0.61
0.180	0.2083	0.0463	0.86	0.56	6.51	0.59	0.126E-03	0.222	1.94	0.59
0.190	0.2167	0.0456	0.88	0.55	6.49	0.51	0.125E-03	0.211	1.92	0.57
0.200	0.2251	0.0450	0.89	0.55	6.47	0.45	0.125E-03	0.200	1.90	0.55
0.220	0.2421	0.0440	0.91	0.55	6.44	0.35	0.123E-03	0.182	1.87	0.52
0.240	0.2592	0.0432	0.93	0.54	6.42	0.27	0.121E-03	0.167	1.85	0.49
0.260	0.2766	0.0426	0.94	0.54	6.40	0.21	0.119E-03	0.154	1.83	0.47
0.280	0.2942	0.0420	0.95	0.54	6.39	0.16	0.118E-03	0.143	1.82	0.46
0.300	0.3121	0.0416	0.96	0.54	6.38	0.13	0.116E-03	0.133	1.81	0.44
0.320	0.3303	0.0413	0.97	0.54	6.38	0.10	0.114E-03	0.125	1.80	0.43
0.340	0.3486	0.0410	0.98	0.53	6.37	0.08	0.113E-03	0.118	1.80	0.43
0.360	0.3672	0.0408	0.98	0.53	6.37	0.06	0.111E-03	0.111	1.79	0.42
0.380	0.3860	0.0406	0.98	0.53	6.37	0.05	0.110E-03	0.105	1.79	0.41
0.400	0.4050	0.0405	0.99	0.53	6.36	0.04	0.109E-03	0.100	1.78	0.41
0.420	0.4241	0.0404	0.99	0.53	6.36	0.03	0.108E-03	0.095	1.78	0.41
0.450	0.4530	0.0403	0.99	0.53	6.36	0.02	0.107E-03	0.089	1.78	0.40
0.500	0.5011	0.0401	1.00	0.53	6.36	0.01	0.106E-03	0.080	1.78	0.40
0.600	0.5994	0.0400	1.00	0.53	6.36	0.00	0.105E-03	0.067	1.77	0.39
0.700	0.6985	0.0399	1.00	0.53	6.36	0.00	0.104E-03	0.057	1.77	0.39
1.000	0.9969	0.0399	1.00	0.53	6.36	0.00	0.104E-03	0.040	1.77	0.39
INFINIT	INFINITE	0.0398	1.00	0.53	6.36	0	0.104E-03	0	1.77	0.39

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

TABLE for PROGRESSIVE CNOIDAL WAVES

5%

H/L_o = 5%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	K(m)	m
0.060	0.0983	0.0819	0.61	0.75	11.14	6.32	0.179E-03	0.833	3.93	0.99
0.070	0.1096	0.0783	0.64	0.71	9.72	4.54	0.184E-03	0.714	3.41	0.98
0.080	0.1202	0.0751	0.67	0.68	8.79	3.42	0.187E-03	0.625	3.05	0.96
0.090	0.1304	0.0725	0.69	0.66	8.15	2.66	0.190E-03	0.556	2.79	0.94
0.100	0.1406	0.0703	0.71	0.64	7.71	2.10	0.192E-03	0.500	2.60	0.91
0.110	0.1496	0.0680	0.74	0.62	7.40	1.73	0.195E-03	0.455	2.47	0.87
0.120	0.1581	0.0659	0.76	0.61	7.19	1.46	0.197E-03	0.417	2.36	0.84
0.130	0.1666	0.0641	0.78	0.60	7.03	1.23	0.199E-03	0.385	2.27	0.81
0.140	0.1750	0.0625	0.80	0.59	6.90	1.05	0.200E-03	0.357	2.21	0.78
0.150	0.1833	0.0611	0.82	0.58	6.81	0.90	0.201E-03	0.333	2.15	0.75
0.160	0.1916	0.0599	0.83	0.58	6.74	0.78	0.201E-03	0.313	2.11	0.72
0.170	0.2000	0.0588	0.85	0.57	6.68	0.68	0.201E-03	0.294	2.07	0.69
0.180	0.2083	0.0579	0.86	0.57	6.64	0.59	0.201E-03	0.278	2.04	0.67
0.190	0.2167	0.0570	0.88	0.56	6.60	0.51	0.200E-03	0.263	2.01	0.65
0.200	0.2251	0.0563	0.89	0.56	6.57	0.45	0.199E-03	0.250	1.99	0.63
0.220	0.2421	0.0550	0.91	0.56	6.52	0.35	0.196E-03	0.227	1.95	0.60
0.240	0.2592	0.0540	0.93	0.55	6.49	0.27	0.193E-03	0.208	1.92	0.57
0.260	0.2766	0.0532	0.94	0.55	6.47	0.21	0.190E-03	0.192	1.90	0.55
0.280	0.2942	0.0525	0.95	0.55	6.45	0.16	0.187E-03	0.179	1.88	0.53
0.300	0.3121	0.0520	0.96	0.55	6.44	0.13	0.185E-03	0.167	1.87	0.52
0.320	0.3303	0.0516	0.97	0.54	6.43	0.10	0.182E-03	0.156	1.86	0.51
0.340	0.3486	0.0513	0.98	0.54	6.42	0.08	0.179E-03	0.147	1.85	0.50
0.360	0.3672	0.0510	0.98	0.54	6.42	0.06	0.177E-03	0.139	1.85	0.49
0.380	0.3860	0.0508	0.98	0.54	6.41	0.05	0.175E-03	0.132	1.84	0.49
0.400	0.4050	0.0506	0.99	0.54	6.41	0.04	0.174E-03	0.125	1.84	0.48
0.420	0.4241	0.0505	0.99	0.54	6.41	0.03	0.172E-03	0.119	1.84	0.48
0.450	0.4523	0.0503	0.99	0.54	6.40	0.02	0.171E-03	0.111	1.83	0.47
0.500	0.5005	0.0501	1.00	0.54	6.40	0.01	0.169E-03	0.100	1.83	0.47
0.600	0.5984	0.0499	1.00	0.54	6.40	0.00	0.167E-03	0.083	1.83	0.47
0.700	0.6973	0.0498	1.00	0.54	6.40	0.00	0.166E-03	0.071	1.82	0.46
1.000	0.9949	0.0497	1.01	0.54	6.40	0.00	0.166E-03	0.050	1.82	0.46
INFINIT	INFINITE	0.0497	1.01	0.54	6.40	0	0.166E-03	0	1.82	0.46

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

6%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 6% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.070	0.1078	0.0924	0.65	0.74	10.66	4.77	0.283E-03	0.857	3.75	0.99
0.080	0.1188	0.0891	0.67	0.71	9.53	3.55	0.284E-03	0.750	3.34	0.98
0.090	0.1295	0.0863	0.70	0.68	8.74	2.72	0.286E-03	0.667	3.03	0.96
0.100	0.1397	0.0838	0.72	0.66	8.19	2.15	0.288E-03	0.600	2.81	0.94
0.110	0.1496	0.0816	0.74	0.64	7.79	1.73	0.289E-03	0.545	2.64	0.91
0.120	0.1581	0.0791	0.76	0.63	7.51	1.46	0.292E-03	0.500	2.52	0.89
0.130	0.1666	0.0769	0.78	0.62	7.30	1.23	0.294E-03	0.462	2.42	0.86
0.140	0.1750	0.0750	0.80	0.61	7.14	1.05	0.296E-03	0.429	2.34	0.83
0.150	0.1833	0.0733	0.82	0.60	7.02	0.90	0.297E-03	0.400	2.27	0.81
0.160	0.1916	0.0719	0.83	0.59	6.92	0.78	0.297E-03	0.375	2.21	0.78
0.170	0.2000	0.0706	0.85	0.59	6.84	0.68	0.297E-03	0.353	2.17	0.76
0.180	0.2083	0.0694	0.86	0.58	6.78	0.59	0.296E-03	0.333	2.13	0.74
0.190	0.2167	0.0684	0.88	0.58	6.73	0.51	0.295E-03	0.316	2.10	0.72
0.200	0.2251	0.0675	0.89	0.57	6.69	0.45	0.293E-03	0.300	2.07	0.70
0.220	0.2421	0.0660	0.91	0.57	6.62	0.35	0.290E-03	0.273	2.03	0.66
0.240	0.2592	0.0648	0.93	0.56	6.58	0.27	0.285E-03	0.250	1.99	0.64
0.260	0.2766	0.0638	0.94	0.56	6.55	0.21	0.281E-03	0.231	1.97	0.62
0.280	0.2942	0.0631	0.95	0.56	6.52	0.16	0.276E-03	0.214	1.95	0.60
0.300	0.3121	0.0624	0.96	0.55	6.50	0.13	0.272E-03	0.200	1.93	0.58
0.320	0.3303	0.0619	0.97	0.55	6.49	0.10	0.268E-03	0.188	1.92	0.57
0.340	0.3486	0.0615	0.98	0.55	6.48	0.08	0.264E-03	0.176	1.91	0.56
0.360	0.3672	0.0612	0.98	0.55	6.47	0.06	0.261E-03	0.167	1.90	0.56
0.380	0.3860	0.0609	0.98	0.55	6.47	0.05	0.258E-03	0.158	1.90	0.55
0.400	0.4044	0.0607	0.99	0.55	6.46	0.04	0.256E-03	0.150	1.89	0.54
0.420	0.4231	0.0604	0.99	0.55	6.46	0.03	0.254E-03	0.143	1.89	0.54
0.450	0.4515	0.0602	1.00	0.55	6.45	0.02	0.252E-03	0.133	1.89	0.54
0.500	0.4996	0.0600	1.00	0.55	6.45	0.01	0.249E-03	0.120	1.88	0.53
0.600	0.5971	0.0597	1.00	0.55	6.45	0.00	0.246E-03	0.100	1.88	0.53
0.700	0.6956	0.0596	1.01	0.55	6.44	0.00	0.245E-03	0.086	1.88	0.53
1.000	0.9921	0.0595	1.01	0.55	6.44	0.00	0.245E-03	0.060	1.88	0.52
INFINIT	INFINITE	0.0594	1.01	0.55	6.44	0	0.246E-03	0	1.87	0.52

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

TABLE for PROGRESSIVE CNOIDAL WAVES

7%

H/L_o = 7% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.080	0.1168	0.1022	0.68	0.73	10.30	3.74	0.414E-03	0.875	3.62	0.99
0.090	0.1277	0.0994	0.70	0.70	9.36	2.83	0.412E-03	0.778	3.27	0.98
0.100	0.1383	0.0968	0.72	0.68	8.69	2.22	0.411E-03	0.700	3.01	0.96
0.110	0.1485	0.0945	0.74	0.66	8.21	1.77	0.410E-03	0.636	2.82	0.94
0.120	0.1581	0.0922	0.76	0.64	7.86	1.46	0.411E-03	0.583	2.67	0.92
0.130	0.1666	0.0897	0.78	0.63	7.60	1.23	0.413E-03	0.538	2.56	0.90
0.140	0.1750	0.0875	0.80	0.62	7.40	1.05	0.415E-03	0.500	2.46	0.87
0.150	0.1833	0.0855	0.82	0.61	7.24	0.90	0.416E-03	0.467	2.39	0.85
0.160	0.1916	0.0838	0.83	0.61	7.12	0.78	0.416E-03	0.438	2.32	0.83
0.170	0.2000	0.0823	0.85	0.60	7.02	0.68	0.415E-03	0.412	2.27	0.81
0.180	0.2083	0.0810	0.86	0.59	6.94	0.59	0.414E-03	0.389	2.23	0.79
0.190	0.2167	0.0798	0.88	0.59	6.87	0.51	0.412E-03	0.368	2.19	0.77
0.200	0.2251	0.0788	0.89	0.58	6.82	0.45	0.410E-03	0.350	2.16	0.75
0.220	0.2421	0.0770	0.91	0.58	6.74	0.35	0.405E-03	0.318	2.10	0.72
0.240	0.2592	0.0756	0.93	0.57	6.68	0.27	0.399E-03	0.292	2.06	0.69
0.260	0.2766	0.0745	0.94	0.57	6.64	0.21	0.392E-03	0.269	2.04	0.67
0.280	0.2942	0.0736	0.95	0.57	6.60	0.16	0.386E-03	0.250	2.01	0.65
0.300	0.3121	0.0728	0.96	0.56	6.58	0.13	0.380E-03	0.233	1.99	0.64
0.320	0.3303	0.0722	0.97	0.56	6.56	0.10	0.374E-03	0.219	1.98	0.63
0.340	0.3486	0.0718	0.98	0.56	6.55	0.08	0.369E-03	0.206	1.97	0.62
0.360	0.3672	0.0714	0.98	0.56	6.54	0.06	0.365E-03	0.194	1.96	0.61
0.380	0.3852	0.0710	0.99	0.56	6.53	0.05	0.361E-03	0.184	1.95	0.60
0.400	0.4036	0.0706	0.99	0.56	6.52	0.04	0.358E-03	0.175	1.95	0.60
0.420	0.4222	0.0704	0.99	0.56	6.52	0.03	0.355E-03	0.167	1.94	0.59
0.450	0.4505	0.0701	1.00	0.56	6.51	0.02	0.352E-03	0.156	1.94	0.59
0.500	0.4983	0.0698	1.00	0.55	6.51	0.01	0.348E-03	0.140	1.93	0.59
0.600	0.5953	0.0695	1.01	0.55	6.50	0.00	0.345E-03	0.117	1.93	0.58
0.700	0.6933	0.0693	1.01	0.55	6.50	0.00	0.344E-03	0.100	1.93	0.58
1.000	0.9884	0.0692	1.01	0.55	6.50	0.00	0.343E-03	0.070	1.93	0.58
INFINIT	INFINITE	0.0690	1.02	0.55	6.50	0	0.344E-03	0	1.93	0.58

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\, T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

8%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 8%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.090	0.1256	0.1117	0.72	0.72	10.00	2.99	0.577E-03	0.889	3.51	0.99
0.100	0.1366	0.1093	0.73	0.70	9.22	2.31	0.570E-03	0.800	3.22	0.97
0.110	0.1470	0.1069	0.75	0.68	8.65	1.83	0.565E-03	0.727	3.00	0.96
0.120	0.1572	0.1048	0.76	0.66	8.23	1.48	0.560E-03	0.667	2.83	0.94
0.130	0.1666	0.1025	0.78	0.65	7.91	1.23	0.559E-03	0.615	2.69	0.92
0.140	0.1749	0.1000	0.80	0.64	7.67	1.05	0.560E-03	0.571	2.59	0.90
0.150	0.1833	0.0978	0.82	0.63	7.48	0.90	0.561E-03	0.533	2.50	0.88
0.160	0.1916	0.0958	0.83	0.62	7.33	0.78	0.560E-03	0.500	2.43	0.86
0.170	0.2000	0.0941	0.85	0.61	7.21	0.68	0.559E-03	0.471	2.37	0.84
0.180	0.2083	0.0926	0.86	0.61	7.11	0.59	0.557E-03	0.444	2.32	0.83
0.190	0.2167	0.0912	0.88	0.60	7.03	0.51	0.555E-03	0.421	2.28	0.81
0.200	0.2251	0.0900	0.89	0.60	6.96	0.45	0.552E-03	0.400	2.24	0.79
0.220	0.2421	0.0880	0.91	0.59	6.86	0.35	0.544E-03	0.364	2.18	0.76
0.240	0.2592	0.0864	0.93	0.58	6.79	0.27	0.536E-03	0.333	2.14	0.74
0.260	0.2766	0.0851	0.94	0.58	6.73	0.21	0.527E-03	0.308	2.10	0.72
0.280	0.2942	0.0841	0.95	0.57	6.70	0.16	0.518E-03	0.286	2.08	0.70
0.300	0.3121	0.0832	0.96	0.57	6.67	0.13	0.510E-03	0.267	2.06	0.69
0.320	0.3303	0.0826	0.97	0.57	6.64	0.10	0.502E-03	0.250	2.04	0.68
0.340	0.3486	0.0820	0.98	0.57	6.63	0.08	0.496E-03	0.235	2.03	0.67
0.360	0.3661	0.0814	0.98	0.57	6.61	0.06	0.491E-03	0.222	2.02	0.66
0.380	0.3842	0.0809	0.99	0.56	6.60	0.05	0.486E-03	0.211	2.01	0.65
0.400	0.4025	0.0805	0.99	0.56	6.59	0.04	0.482E-03	0.200	2.00	0.65
0.420	0.4210	0.0802	1.00	0.56	6.58	0.03	0.479E-03	0.190	2.00	0.64
0.450	0.4491	0.0798	1.00	0.56	6.58	0.02	0.474E-03	0.178	1.99	0.64
0.500	0.4965	0.0794	1.01	0.56	6.57	0.01	0.469E-03	0.160	1.99	0.63
0.600	0.5930	0.0791	1.01	0.56	6.56	0.00	0.465E-03	0.133	1.98	0.63
0.700	0.6903	0.0789	1.01	0.56	6.56	0.00	0.463E-03	0.114	1.98	0.63
1.000	0.9837	0.0787	1.02	0.56	6.56	0.00	0.463E-03	0.080	1.98	0.62
INFINIT	INFINITE	0.0784	1.02	0.56	6.56	0	0.465E-03	0	1.98	0.62

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

9%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 9% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.110	0.1449	0.1186	0.76	0.69	9.11	1.91	0.760E-03	0.818	3.18	0.97
0.120	0.1553	0.1165	0.77	0.68	8.62	1.54	0.749E-03	0.750	2.98	0.96
0.130	0.1654	0.1145	0.79	0.66	8.24	1.26	0.741E-03	0.692	2.83	0.94
0.140	0.1749	0.1125	0.80	0.65	7.96	1.05	0.736E-03	0.643	2.71	0.92
0.150	0.1833	0.1100	0.82	0.64	7.74	0.90	0.735E-03	0.600	2.62	0.91
0.160	0.1916	0.1078	0.83	0.63	7.56	0.78	0.734E-03	0.562	2.54	0.89
0.170	0.2000	0.1059	0.85	0.62	7.42	0.68	0.731E-03	0.529	2.47	0.87
0.180	0.2083	0.1042	0.86	0.62	7.30	0.59	0.728E-03	0.500	2.42	0.86
0.190	0.2167	0.1027	0.88	0.61	7.20	0.51	0.725E-03	0.474	2.37	0.84
0.200	0.2251	0.1013	0.89	0.61	7.12	0.45	0.720E-03	0.450	2.33	0.83
0.220	0.2421	0.0990	0.91	0.60	7.00	0.35	0.710E-03	0.409	2.26	0.80
0.240	0.2592	0.0972	0.93	0.59	6.91	0.27	0.699E-03	0.375	2.21	0.78
0.260	0.2766	0.0957	0.94	0.59	6.84	0.21	0.688E-03	0.346	2.17	0.76
0.280	0.2942	0.0946	0.95	0.58	6.80	0.16	0.676E-03	0.321	2.14	0.74
0.300	0.3121	0.0936	0.96	0.58	6.76	0.13	0.665E-03	0.300	2.12	0.73
0.320	0.3299	0.0928	0.97	0.58	6.73	0.10	0.656E-03	0.281	2.10	0.72
0.340	0.3472	0.0919	0.98	0.58	6.71	0.08	0.648E-03	0.265	2.09	0.71
0.360	0.3653	0.0913	0.99	0.57	6.69	0.06	0.642E-03	0.250	2.07	0.70
0.380	0.3833	0.0908	0.99	0.57	6.68	0.05	0.636E-03	0.237	2.07	0.69
0.400	0.4015	0.0903	1.00	0.57	6.67	0.04	0.631E-03	0.225	2.06	0.69
0.420	0.4199	0.0900	1.00	0.57	6.66	0.03	0.626E-03	0.214	2.05	0.68
0.450	0.4478	0.0896	1.01	0.57	6.65	0.02	0.621E-03	0.200	2.04	0.68
0.500	0.4949	0.0891	1.01	0.57	6.64	0.01	0.615E-03	0.180	2.04	0.67
0.600	0.5907	0.0886	1.02	0.57	6.63	0.00	0.609E-03	0.150	2.03	0.67
0.700	0.6874	0.0884	1.02	0.57	6.63	0.00	0.607E-03	0.129	2.03	0.67
1.000	0.9792	0.0881	1.02	0.57	6.62	0.00	0.607E-03	0.090	2.03	0.67
INFINIT	INFINITE	0.0876	1.03	0.57	6.62	0	0.609E-03	0	2.02	0.66

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

10%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 10% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.120	0.1529	0.1274	0.78	0.69	9.02	1.62	0.983E-03	0.833	3.14	0.97
0.130	0.1633	0.1256	0.80	0.67	8.59	1.31	0.967E-03	0.769	2.97	0.96
0.140	0.1736	0.1240	0.81	0.66	8.26	1.08	0.955E-03	0.714	2.84	0.94
0.150	0.1833	0.1222	0.82	0.65	8.00	0.90	0.944E-03	0.667	2.73	0.93
0.160	0.1916	0.1198	0.83	0.64	7.80	0.78	0.940E-03	0.625	2.64	0.91
0.170	0.2000	0.1176	0.85	0.63	7.63	0.68	0.936E-03	0.588	2.57	0.90
0.180	0.2083	0.1157	0.86	0.63	7.50	0.59	0.931E-03	0.556	2.51	0.88
0.190	0.2167	0.1141	0.88	0.62	7.38	0.51	0.926E-03	0.526	2.46	0.87
0.200	0.2251	0.1126	0.89	0.62	7.29	0.45	0.920E-03	0.500	2.41	0.86
0.220	0.2421	0.1100	0.91	0.61	7.14	0.35	0.907E-03	0.455	2.34	0.83
0.240	0.2592	0.1080	0.93	0.60	7.04	0.27	0.892E-03	0.417	2.28	0.81
0.260	0.2766	0.1064	0.94	0.60	6.96	0.21	0.877E-03	0.385	2.24	0.79
0.280	0.2942	0.1051	0.95	0.59	6.90	0.16	0.862E-03	0.357	2.21	0.78
0.300	0.3116	0.1039	0.96	0.59	6.86	0.13	0.849E-03	0.333	2.18	0.76
0.320	0.3289	0.1028	0.97	0.59	6.83	0.10	0.839E-03	0.312	2.16	0.75
0.340	0.3463	0.1019	0.98	0.58	6.80	0.08	0.830E-03	0.294	2.14	0.74
0.360	0.3640	0.1011	0.99	0.58	6.78	0.06	0.821E-03	0.278	2.13	0.73
0.380	0.3818	0.1005	1.00	0.58	6.76	0.05	0.814E-03	0.263	2.12	0.73
0.400	0.3999	0.1000	1.00	0.58	6.75	0.04	0.807E-03	0.250	2.11	0.72
0.420	0.4181	0.0995	1.00	0.58	6.74	0.03	0.801E-03	0.238	2.10	0.72
0.450	0.4457	0.0991	1.01	0.58	6.73	0.02	0.795E-03	0.222	2.10	0.71
0.500	0.4924	0.0985	1.02	0.58	6.71	0.01	0.787E-03	0.200	2.09	0.71
0.600	0.5874	0.0979	1.02	0.57	6.70	0.00	0.779E-03	0.167	2.08	0.70
0.700	0.6834	0.0976	1.02	0.57	6.70	0.00	0.777E-03	0.143	2.08	0.70
1.000	0.9729	0.0973	1.03	0.57	6.69	0.00	0.777E-03	0.100	2.08	0.70
INFINIT	INFINITE	0.0966	1.04	0.57	6.69	0	0.781E-03	0	2.07	0.70

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

11%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 11%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.130	0.1606	0.1359	0.81	0.69	8.96	1.38	0.124E-02	0.846	3.12	0.97
0.140	0.1711	0.1345	0.82	0.67	8.57	1.13	0.122E-02	0.786	2.97	0.96
0.150	0.1810	0.1327	0.83	0.66	8.28	0.94	0.120E-02	0.733	2.85	0.94
0.160	0.1907	0.1311	0.84	0.65	8.04	0.79	0.119E-02	0.687	2.75	0.93
0.170	0.2000	0.1294	0.85	0.64	7.85	0.68	0.118E-02	0.647	2.67	0.92
0.180	0.2083	0.1273	0.86	0.64	7.70	0.59	0.117E-02	0.611	2.60	0.90
0.190	0.2167	0.1255	0.88	0.63	7.57	0.51	0.116E-02	0.579	2.54	0.89
0.200	0.2251	0.1238	0.89	0.63	7.46	0.45	0.115E-02	0.550	2.49	0.88
0.220	0.2421	0.1210	0.91	0.62	7.30	0.35	0.114E-02	0.500	2.41	0.86
0.240	0.2592	0.1188	0.93	0.61	7.18	0.27	0.112E-02	0.458	2.35	0.84
0.260	0.2766	0.1170	0.94	0.60	7.09	0.21	0.110E-02	0.423	2.31	0.82
0.280	0.2936	0.1154	0.95	0.60	7.02	0.17	0.108E-02	0.393	2.27	0.81
0.300	0.3105	0.1139	0.97	0.60	6.97	0.13	0.107E-02	0.367	2.24	0.79
0.320	0.3276	0.1126	0.98	0.59	6.92	0.10	0.105E-02	0.344	2.22	0.78
0.340	0.3449	0.1116	0.99	0.59	6.89	0.08	0.104E-02	0.324	2.20	0.77
0.360	0.3623	0.1107	0.99	0.59	6.87	0.07	0.103E-02	0.306	2.19	0.77
0.380	0.3800	0.1100	1.00	0.59	6.85	0.05	0.102E-02	0.289	2.17	0.76
0.400	0.3979	0.1094	1.01	0.59	6.83	0.04	0.101E-02	0.275	2.16	0.75
0.420	0.4159	0.1089	1.01	0.58	6.82	0.03	0.101E-02	0.262	2.16	0.75
0.450	0.4433	0.1084	1.02	0.58	6.81	0.02	0.999E-03	0.244	2.15	0.75
0.500	0.4896	0.1077	1.02	0.58	6.79	0.01	0.989E-03	0.220	2.14	0.74
0.600	0.5837	0.1070	1.03	0.58	6.78	0.00	0.980E-03	0.183	2.13	0.73
0.700	0.6788	0.1067	1.03	0.58	6.77	0.00	0.977E-03	0.157	2.13	0.73
1.000	0.9658	0.1062	1.04	0.58	6.77	0.00	0.978E-03	0.110	2.12	0.73
INFINIT	INFINITE	0.1053	1.04	0.58	6.76	0	0.983E-03	0	2.12	0.73

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 12% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.140	0.1681	0.1440	0.83	0.69	8.91	1.20	0.154E-02	0.857	3.10	0.97
0.150	0.1785	0.1428	0.84	0.67	8.57	0.99	0.152E-02	0.800	2.96	0.96
0.160	0.1884	0.1413	0.85	0.66	8.30	0.83	0.149E-02	0.750	2.86	0.94
0.170	0.1978	0.1396	0.86	0.65	8.09	0.70	0.147E-02	0.706	2.77	0.93
0.180	0.2072	0.1382	0.87	0.65	7.91	0.60	0.146E-02	0.667	2.69	0.92
0.190	0.2163	0.1366	0.88	0.64	7.76	0.52	0.144E-02	0.632	2.63	0.91
0.200	0.2249	0.1349	0.89	0.63	7.64	0.45	0.143E-02	0.600	2.58	0.90
0.220	0.2421	0.1320	0.91	0.63	7.46	0.35	0.140E-02	0.545	2.49	0.88
0.240	0.2589	0.1294	0.93	0.62	7.32	0.27	0.138E-02	0.500	2.42	0.86
0.260	0.2753	0.1270	0.94	0.61	7.21	0.21	0.136E-02	0.462	2.37	0.84
0.280	0.2918	0.1251	0.96	0.61	7.14	0.17	0.134E-02	0.429	2.33	0.83
0.300	0.3085	0.1234	0.97	0.60	7.07	0.14	0.132E-02	0.400	2.30	0.82
0.320	0.3254	0.1220	0.98	0.60	7.03	0.11	0.131E-02	0.375	2.28	0.81
0.340	0.3425	0.1209	0.99	0.60	6.99	0.09	0.129E-02	0.353	2.26	0.80
0.360	0.3598	0.1199	1.00	0.60	6.96	0.07	0.128E-02	0.333	2.24	0.79
0.380	0.3773	0.1191	1.01	0.59	6.94	0.05	0.127E-02	0.316	2.23	0.79
0.400	0.3950	0.1185	1.01	0.59	6.92	0.04	0.126E-02	0.300	2.22	0.78
0.420	0.4129	0.1180	1.02	0.59	6.90	0.03	0.125E-02	0.286	2.21	0.78
0.450	0.4400	0.1173	1.02	0.59	6.89	0.02	0.124E-02	0.267	2.20	0.77
0.500	0.4858	0.1166	1.03	0.59	6.87	0.01	0.122E-02	0.240	2.19	0.77
0.600	0.5794	0.1159	1.04	0.59	6.85	0.00	0.121E-02	0.200	2.18	0.76
0.700	0.6736	0.1155	1.04	0.59	6.85	0.00	0.121E-02	0.171	2.17	0.76
1.000	0.9579	0.1149	1.04	0.59	6.84	0.00	0.121E-02	0.120	2.17	0.76
INFINIT	INFINITE	0.1138	1.05	0.59	6.83	0	0.122E-02	0	2.16	0.75

L_o and c_o are first order sinusoidal deep water values (L_o = 1.56 T^2 metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

13%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 13%   (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	K(m)	m
0.150	0.1754	0.1520	0.86	0.68	8.87	1.04	0.189E-02	0.867	3.08	0.97
0.160	0.1853	0.1505	0.86	0.67	8.57	0.87	0.185E-02	0.813	2.96	0.96
0.170	0.1951	0.1492	0.87	0.66	8.33	0.74	0.182E-02	0.765	2.87	0.95
0.180	0.2047	0.1479	0.88	0.66	8.13	0.63	0.180E-02	0.722	2.78	0.94
0.190	0.2139	0.1463	0.89	0.65	7.97	0.54	0.178E-02	0.684	2.72	0.93
0.200	0.2226	0.1447	0.90	0.64	7.83	0.47	0.176E-02	0.650	2.66	0.92
0.220	0.2402	0.1419	0.92	0.63	7.62	0.36	0.172E-02	0.591	2.56	0.90
0.240	0.2570	0.1392	0.93	0.63	7.46	0.28	0.169E-02	0.542	2.49	0.88
0.260	0.2736	0.1368	0.95	0.62	7.35	0.22	0.167E-02	0.500	2.44	0.87
0.280	0.2901	0.1347	0.97	0.61	7.26	0.17	0.164E-02	0.464	2.39	0.85
0.300	0.3067	0.1329	0.98	0.61	7.19	0.14	0.162E-02	0.433	2.36	0.84
0.320	0.3234	0.1314	0.99	0.61	7.13	0.11	0.160E-02	0.406	2.33	0.83
0.340	0.3403	0.1301	1.00	0.60	7.09	0.09	0.158E-02	0.382	2.31	0.82
0.360	0.3571	0.1290	1.01	0.60	7.06	0.07	0.156E-02	0.361	2.29	0.81
0.380	0.3744	0.1281	1.01	0.60	7.03	0.06	0.155E-02	0.342	2.28	0.81
0.400	0.3919	0.1274	1.02	0.60	7.01	0.04	0.154E-02	0.325	2.27	0.80
0.420	0.4096	0.1268	1.02	0.60	6.99	0.04	0.153E-02	0.310	2.26	0.80
0.450	0.4364	0.1261	1.03	0.60	6.97	0.03	0.151E-02	0.289	2.25	0.79
0.500	0.4817	0.1252	1.04	0.59	6.95	0.01	0.150E-02	0.260	2.23	0.79
0.600	0.5738	0.1243	1.05	0.59	6.93	0.00	0.148E-02	0.217	2.22	0.78
0.700	0.6668	0.1238	1.05	0.59	6.92	0.00	0.148E-02	0.186	2.22	0.78
1.000	0.9476	0.1232	1.05	0.59	6.92	0.00	0.148E-02	0.130	2.21	0.78
INFINIT	INFINITE	0.1219	1.07	0.59	6.90	0	0.149E-02	0	2.21	0.78

L_o and c_o are first order sinusoidal deep water values (L_o = 1.56 T^2 metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

14%

TABLE for PROGRESSIVE CNOIDAL WAVES

H/L_o = 14% (first order sinusoidal deep water steepness)

D/L_o	D/L	H/L	c/c_o	η_c/H	rL	β	$\overline{E_{flux}}/\gamma c_o L_o^2$	H/D	$K(m)$	m
0.190	0.2111	0.1555	0.90	0.66	8.17	0.56	0.217E-02	0.737	2.80	0.94
0.200	0.2200	0.1540	0.91	0.65	8.02	0.49	0.214E-02	0.700	2.74	0.93
0.220	0.2377	0.1513	0.93	0.64	7.78	0.37	0.210E-02	0.636	2.64	0.91
0.240	0.2546	0.1485	0.94	0.63	7.61	0.29	0.206E-02	0.583	2.56	0.90
0.260	0.2712	0.1460	0.96	0.63	7.48	0.23	0.202E-02	0.538	2.50	0.88
0.280	0.2874	0.1437	0.97	0.62	7.38	0.18	0.199E-02	0.500	2.45	0.87
0.300	0.3040	0.1419	0.99	0.62	7.30	0.14	0.196E-02	0.467	2.42	0.86
0.320	0.3207	0.1403	1.00	0.61	7.24	0.11	0.194E-02	0.437	2.39	0.85
0.340	0.3377	0.1391	1.01	0.61	7.20	0.09	0.191E-02	0.412	2.36	0.84
0.360	0.3546	0.1379	1.02	0.61	7.16	0.07	0.189E-02	0.389	2.34	0.83
0.380	0.3717	0.1370	1.02	0.61	7.13	0.06	0.187E-02	0.368	2.33	0.83
0.400	0.3890	0.1362	1.03	0.60	7.10	0.05	0.186E-02	0.350	2.32	0.82
0.420	0.4065	0.1355	1.03	0.60	7.08	0.04	0.185E-02	0.333	2.30	0.82
0.450	0.4329	0.1347	1.04	0.60	7.06	0.03	0.183E-02	0.311	2.29	0.82
0.500	0.4772	0.1336	1.05	0.60	7.03	0.02	0.181E-02	0.280	2.28	0.81
0.600	0.5681	0.1326	1.06	0.60	7.01	0.00	0.179E-02	0.233	2.27	0.80
0.700	0.6600	0.1320	1.06	0.60	7.00	0.00	0.179E-02	0.200	2.26	0.80
1.000	0.9374	0.1312	1.07	0.60	6.99	0.00	0.179E-02	0.140	2.26	0.80
INFINIT	INFINITE	0.1297	1.08	0.60	6.98	0	0.180E-02	0	2.25	0.80

L_o and c_o are first order sinusoidal deep water values ($L_o = 1.56\,T^2$ metres)

$\overline{E_{flux}}$ is the mean value over a wave period T of the energy flux

NUMERICAL EXAMPLE

To show the practical use of the formulas and the table a numerical example with a given wave on infinite deep water is considered. The same wave is then followed as it approaches the coast, until the breaker height.

Let us consider a wave with the period T = 1o seconds and the deep water wave height H_o = 3.12 meters. The sinusoidal deep water wave length will then be

$$L_0 = \frac{g}{2\pi} T^2 = 1.56 \cdot 10^2 = 156 \text{ m} \tag{1}$$

We then get the sinusoidal deep water steepness

$$\frac{H_o}{L_o} = \frac{3.12}{156} = 0.02 = 2\ \% \tag{2}$$

For the sinusoidal deep water celerity we get

$$c_o = \frac{L_o}{T} = \frac{156}{10} = 15.6 \text{ m/sec.} \tag{3}$$

We now use the table with H/L_o = o.o2. We see that for D/L_o infinite we have c/c_o = 1.oo, so that the cnoidal deep water celerity and wave length are the same as the sinusoidal within the accuracy used here. (We would get the same results using a third order sinusoidal theory for the same wave here). We find η_c/H = o.52, so that we for the deep water crest height η_c and trough depth η_t will get

$$\eta_c = 0.52 \cdot 3.12 = 1.62 \text{ m} \tag{4}$$

$$|\eta_t| = 0.48 \cdot 3.12 = 1.50 \text{ m} \tag{5}$$

From the table we get rL = 6.3o, so that

$$r = \frac{6.30}{156} = 4.04 \cdot 10^{-2} \text{ m}^{-1} \tag{6}$$

We then get the horizontal particle velocities at the surface of the crest and the trough from eq. D12.

$$u_{cs} = c\,\eta_c\, r = 15.6 \cdot 1.62 \cdot 4.04 \cdot 10^{-2} = 1.02 \text{ m/sec} \tag{7}$$

$$|u_{ts}| = c\,|\eta_t|\, r = 15.6 \cdot 1.50 \cdot 4.04 \cdot 10^{-2} = 0.95 \text{ m/sec} \tag{8}$$

This wave has though got a second order 'backward' rotation. For an irrotational wave we must chose $\delta = 1/2$ in eq. D19. At the surface we get

$$\delta c \left(\frac{H}{2}\right)^2 r^2 = \frac{1}{2}\, 15.6 \left(\frac{3.12}{2}\right)^2 \cdot (4.04 \cdot 10^{-2})^2 = 0.03\ \mathrm{m/sec} \tag{9}$$

Then the particle velocities at the surface of an irrotational wave will be

$$u_{cs} = 1.02 + 0.03 = 1.05\ \mathrm{m/sec.} \tag{10}$$

$$|u_{ts}| = 0.95 - 0.03 = 0.92\ \mathrm{m/sec.} \tag{11}$$

At the depth of 1o m below still water level, SWL, the corresponding u for the irrotational wave will be

$$\begin{aligned} u_{c10} &= c\,\eta_c\, r\, e^{r(-10-\eta_c)} + \delta c \left(\frac{H}{2}\right)^2 r^2 e^{r(-10-\eta_c)} \\ &= 1.02 \cdot 0.63 + 0.02 = 0.66\ \mathrm{m/sec.} \end{aligned} \tag{12}$$

$$\begin{aligned} |u_{t10}| &= c|\eta_t|\, r\, e^{r(-10+|\eta_t|)} - \delta c \left(\frac{H}{2}\right)^2 r^2 e^{r(-10+|\eta_t|)} \\ &= 0.95 \cdot 0.71 - 0.02 = 0.65\ \mathrm{m/sec.} \end{aligned} \tag{13}$$

If the wave is wind made it will be with a 'foreward' rotation, which can be considered by using a bigger value of δ. Rotation will not affect w and p in eqs. D13 and D14.

Let us then consider the pressure below the crest by, eq. D14. We have two equations to use. The first one is the assymptotic value of eq. F14. The last one is much easier to use and will obviously give the wave pressure = o at infinite depth. The two equations are the same except for third order differences. They both give $p = o$ at the surface $z = y$. Using the second equation for the pressure, eq. D14a we find the pressure below the crest at the depth of 1o m below SWL.

$$e^{k(y-z)} = e^{(2\pi/156)(-10-1.62)} = 0.63 \tag{14}$$

$$\frac{p}{\gamma} = 10 + 1.62 \cdot 0.63 + \frac{\pi}{4} \cdot 3.12 \cdot 0.02\,[0.63 - 0.63^2]$$
$$= 10 + 1.03 + 0.01 = 11.04 \text{ m} \tag{15}$$

It is obvious that the last term plays a minor roll. The wave pressure above hydrostatic pressure from SWL will be

$$\frac{p^+}{\gamma} = 11.04 - 10 = 1.04 \text{ m} \tag{16}$$

Let us then use the first equation, eq D14 for the pressure. At the crest $\partial\eta/\partial x = 0$ and $cn^2\theta = 1$. From the table we get $K = 1.67$ and $m = 0.12$. Eq. F16 then gives

$$\frac{\partial^2\eta}{\partial x^2} = -8 \cdot \frac{3.12}{156^2} \cdot 1.67^2\,[0.12 - 1 - 2(2 \cdot 0.12 - 1) \cdot 1 + 3 \cdot 0.12 \cdot 1]$$
$$= -0.0029 \tag{17}$$

We then find the pressure below the crest at the depth of 10 m below SWL

$$\frac{p}{\gamma} = 10 + 1.62 + \frac{15.6^2}{9.81}\Big\{\Big[-0.0029\,\frac{1}{4.04 \cdot 10^{-2}}$$
$$- 1.62(-0.0029)\Big][1 - 0.63] - \frac{1}{2} \cdot 1.62(-0.0029)$$
$$\cdot [1 - 0.39]\Big\} = 11.04 \text{ m} \tag{18}$$

We will now let this wave move ashore without loss of energy, so that the transported energy will stay constant, or the mean energy flux, given in the table, will stay constant. We see from the table that as D/L_o becomes smaller the energy flux will first become bigger and then smaller. This means that first the wave height will decrease and then increase, as also known from sinusoidal waves. For a given depth, D, we find D/L_o. Then we could interpolate to find the value of H/L_o that has got the same value of E_{flux} as the original deep water wave.

But we find such an interpolation rather uncertain in determining H. So we will proceed in a different way shown in appendix II.

We use

$$\frac{H}{H_0} = \sqrt{\frac{E_{flux0}}{E_{flux}}} \tag{19}$$

For a depth of D = 15.6 m we get D/L_o = 15.6/156 = o.1 , and then from the table

$$H = H_0 \sqrt{\frac{E_{flux0}}{E_{flux}}} = 3.12 \sqrt{\frac{253}{289}} = 3.12 \cdot 0.94 = 2.92\,m \tag{20}$$

When we get to a depth of D = 9 m, i.e. D/L_o = 9/156 = o.o58, we have $E_{flux} = E_{flux,o}$, so then $H = H_o$ = 3.12 m. From the table we get

$$\frac{L}{L_0} = \frac{c}{c_0} = 0.58 \tag{21}$$

$$\frac{\eta_c}{H} = 0.64 \tag{22}$$

$$\frac{H}{D} = 0.35 = \frac{3.12}{9} \tag{23}$$

$$rL = 7.80 \tag{24}$$

$$\beta = 6.2 \tag{25}$$

We then get

$$L = 0.58 \cdot 156 = 90\,m \tag{26}$$

$$\eta_c = 0.64 \cdot 3.12 = 2.00\,m \tag{27}$$

$$r = \frac{7.80}{90} = 0.087\ m^{-1} \tag{28}$$

$$5\beta k^3/r^2 = 5 \cdot 6.2 \cdot 0.070^3/0.087^2 = 1.39\ m^{-1} \tag{29}$$

So that for the crest we get

$$R_c = r\left(1 + \tfrac{1}{5}\tanh 5\beta \frac{k^3}{r^2}\eta_c\right) = 0.087\left(1 + \tfrac{1}{5}\tanh(1.39 \cdot 2.00)\right) = 0.104\ m^{-1} \tag{30}$$

and for the trough

$$R_t = 0.087\left(1 - \tfrac{1}{5}\tanh 1.39 \cdot 1.12\right) = 0.071\ m^{-1} \tag{31}$$

If we follow the wave a little further we find that for $D = 4.6$ m we have $H/D = 0.8$. This means that here the wave can be expected to break. As it can be seen on the figure the wave is still far from being solitary in shape.

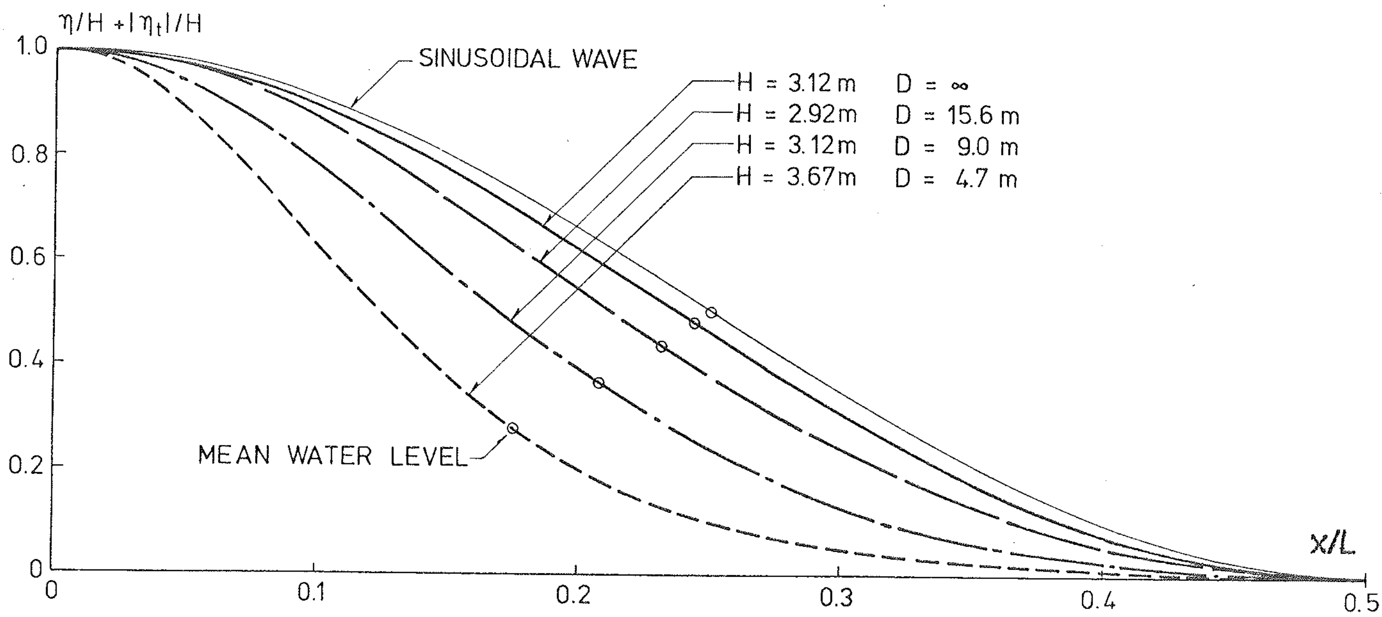

Fig. 3. The change of the wave profile for a cnoidal wave with the constant period $T = 10$ seconds as it proceeds from infinite depth to shallow water without loss of energy.

After having considered this numerical example and after the considerations in the previous chapters we may try to simplify some of the expressions.

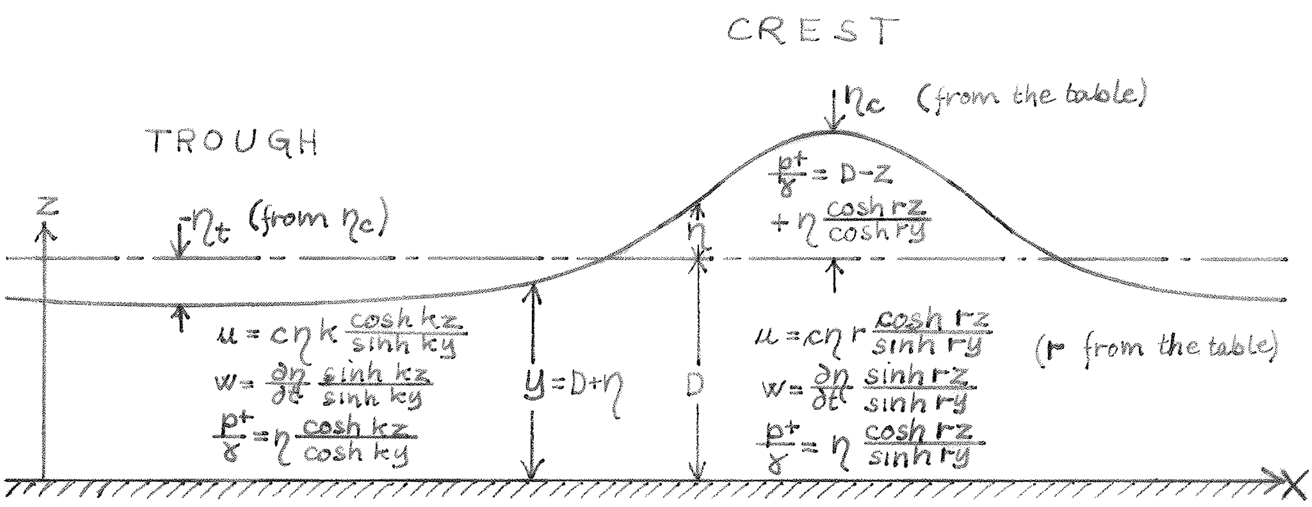

Fig. 4. Simplified expressions in a cnoidal wave. The surface profile (and H, L, and c) are determined by aid of the table. For u, w, and p the simple well known Airy expressions are used, substituting D with y (the actual water depth instead of the mean water depth) and using r (from the table) instead of k below the crest.

With many years of experience with the classical wave theories it may be difficult for the engineer to change to this new theory which he may not find so easy to foresee the results of in every detail. For this reason a combination of the Airy theory and the cnoidal theory can be proposed as shown in fig. 4. This will also make the formulas easier to use.

The wave profile is taken to be cnoidal, while u, w, and p are found by the first order expressions discussed in chapter IV. this will give good practical results, but hydrodynamically the expressions can only be claimed to be of first order.

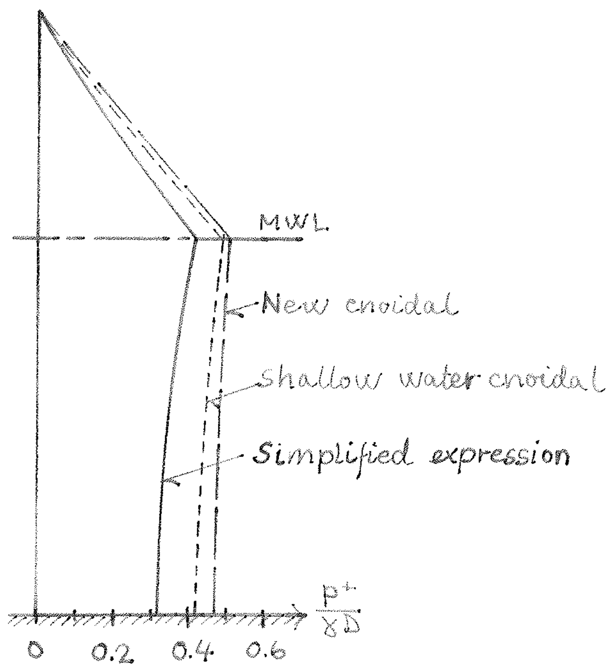

Fig. 5. The simplified expression for the pressure used for the extreme case of a solitary wave, H/D = o.6, compared to the expressions of chapter VIII and chapter IX.

APPENDIX II

PRACTICAL USE OF ENERGY FLUX

It will here be shown how to find the wave height at a given depth from a given deep water wave.

For the sinusoidal wave the mean energy flux is given by

$$E_{flux} = \frac{1}{16} \gamma H^2 c \left(1 + \frac{2kD}{\sinh 2kD}\right) \tag{32}$$

so we see that

$$E_{flux} = c H^2 f(D/L_o) \tag{33}$$

where f is a function.

For the cnoidal wave the situation may seem more complicated, but investigating eq. T18 we see that we have the same simple relation also here. Further we know from chapter IX that in the second order theory the celerity does not depend on the wave height H. The little (third order) dependence on H we have for c in eq. F10 is included of special reasons, and will be negligible here.

So for the cnoidal wave we find

$$E_{flux} = H^2 f(D/L_o) \tag{34}$$

So for two waves with the same preiod, but with different wave heights H_1 and H_2, we find at a given depth (same D/L_o)

$$\frac{H_1}{H_2} = \sqrt{\frac{E_{flux,1}}{E_{flux,2}}} \tag{35}$$

The table uses H/L_o as the main parameter, so for a given period (and then given L_o) a wave of constant wave height, H_o, is considered all over the page. At the given depth $D (\Rightarrow D/L_o)$ the table gives E_{flux} for the wave with the wave height H_o, but we want to find the unknown wave height H for the wave that has the known $E_{flux,o}$. From eq. 35 we see that we simply get

$$\frac{H}{H_o} = \sqrt{\frac{E_{flux,o}}{E_{flux}}} \tag{36}$$

CONCLUSION

Given : T and H_o at infinite depth.

Wanted : H at given D.

Calculate : L_o, H_o/L_o, D/L_o.

Use the table for H_o/L_o and D/L_o to find E_{flux} and to find $E_{flux,o}$.

Use eq. 36.

Change to the table for H/L_o.

Given : T, H, and D.

Wanted : H_o at infinite depth.

Calculate : L_o, H/L_o, D/L_o

Use the table for H/L_o and D/L_o to find E_{flux} and to find $E_{flux,o}$.

Use eq. 36.

Change to the table for H_o/L_o.

Given : T, H_1, and D_1.

Wanted : H_2 at given D_2.

Calculate : L_o, H_1/L_o, D_1/L_o.

Use the table for H_1/L_o and D_1/L_o to find $E_{flux,1}$.

Use the table for H_1/L_o and D_2/L_o to find $E_{flux,2}$.

Use eq. 36 (with substitution of indices).

Change to the table for H_2/L_o.

APPENDIX III

THEORETICAL TEST OF THE DEEP WATER CNOIDAL WAVE

It is shown that the final formulas given for the progressive cnoidal wave on infinite depth fulfil all the hydrodynamic conditions within at least second order approximation.

INTRODUCTION

It may in some cases be difficult to see that the described procedure for finding wave solutions gives correct results. So let us first be convinced that the final cnoidal solutions to η, u, w, and p fulfil the hydrodynamic conditions for an ideal fluid to the second order of approximation. We will examine the kinematics at infinite depth, the boundary conditions at the surface, the equation of continuity, the mass transport, the dynamic equations, and the rotation. We will use the expressions given in this chapter. This means that $z = o$ is at the bottom at infinite depth (and not at the mean water level, as used in chapter VI). It should be noted that it is not necessary to examine e.g. the rotation when the other conditions are fulfilled.

AT INFINITE DEPTH

For finite depth we would demand the vertical velocity at the bottom to be $w = o$. For the waves here we want w to vanish at infinite depth. This is also seen to be the case from eq. D13. We see from eq. D12 that u will also vanish. We also want the wave pressure to vanish. This is also easily seen from eq. D14a.

THE KINEMATIC SURFACE CONDITION

From eq. F1 we get

$$\frac{\partial \eta}{\partial t} = -c\frac{\partial \eta}{\partial x} \tag{37}$$

which is known from any progressive wave. For the surface, $z = y$, the horizontal particle velocity from eq. D12 will be

$$u_s = u = c\eta r \tag{38}$$

Eq. D13 gives the vertical particle velocity at the surface, z = y, as

$$w_s = w = -c\frac{\partial\eta}{\partial x} + c\eta r\frac{\partial\eta}{\partial x} = \frac{\partial\eta}{\partial t} + u_s\frac{\partial\eta}{\partial x} \qquad (39)$$

This shows that a particle on the surface stays on the surface. It should be noted that this condition is fulfilled exactly, despite it needed only to be fulfilled to the second order of approximation like for the Stokes' wave.

THE DYNAMIC SURFACE CONDITION

At the surface of water without surface tension we have only atmospheric pressure, so that the pressure from the water is $p = 0$. From eq. D14 we see that for $z = y$ we get exactly $p = 0$.

THE MASS TRANSPORT

The mean water level is determined so that

$$\int_0^L \eta\, dx = 0 \qquad (40)$$

This condition was used to determine η_t and η_c in eqs. F2 and F3. The water discharge through a vertical will be

$$q = \int_0^y u\, dz = \int_0^y c\eta r e^{r(z-y)} dz = c\eta \qquad (41)$$

where we used eq. D12, with $z = 0$ at infinite depth. The resulting mass transport is then

$$\int_0^T q\, dt = c\int_0^T \eta\, dt = \int_0^L \eta\, dx = 0 \qquad (42)$$

as wanted in a pure wave.

THE EQUATION OF CONTINUITY

From eq. D12 we get

$$\frac{\partial u}{\partial x} = c\left[\frac{\partial \eta}{\partial x} - \eta \frac{\partial \eta}{\partial x} r\right] r e^{r(z-y)} \tag{43}$$

and from eq. D13

$$\frac{\partial w}{\partial z} = c \frac{\partial \eta}{\partial x}\left[-1 + \eta r\right] r e^{r(z-y)} \tag{44}$$

so that we get

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{45}$$

which means that the equation of continuity is fulfilled exactly.

VERTICAL DYNAMIC EQUATION

So far we have fulfilled the conditions exactly. From now we will be content with an approximate fulfillment, so that we will neglect terms that are small of third and higher order in H/L or the equivalent rH. From eq. D14 we get

$$\frac{\partial p}{\partial z} = -\gamma - \rho c^2 \left\{\left[\frac{\partial^2 \eta}{\partial x^2} \frac{1}{r} - \left[2\left(\frac{\partial \eta}{\partial x}\right)^2 + \eta \frac{\partial^2 \eta}{\partial x^2}\right]\right] r e^{r(z-y)} + \left[\left(\frac{\partial \eta}{\partial x}\right)^2 - \eta \frac{\partial^2 \eta}{\partial x^2}\right] r e^{2r(z-y)}\right\} \tag{46}$$

Eq. D13 and D12 give, neglecting third and higher order terms

$$\frac{\partial w}{\partial t} = -c^2 \left[\frac{\partial^2 \eta}{\partial x^2}\left[-1 + \eta r\right] + 2\left(\frac{\partial \eta}{\partial x}\right)^2 r\right] e^{r(z-y)} \tag{47}$$

$$u \frac{\partial w}{\partial x} = -c^2 \eta \frac{\partial^2 \eta}{\partial x^2} r e^{2r(z-y)} \tag{48}$$

$$w \frac{\partial w}{\partial z} = c^2 \left(\frac{\partial \eta}{\partial x}\right)^2 r e^{2r(z-y)} \tag{49}$$

From this we get

$$\frac{\partial p}{\partial z} = -\gamma - \rho\left[\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z}\right] \tag{50}$$

so that the vertical dynamic equation for a frictionless fluid is fulfilled to the second order of approximation.

HORIZONTAL DYNAMIC EQUATION

r from eq. F7 can with eq. D4 be written as

$$r^2 = \frac{16K^2}{L^2}\left(1-\frac{m}{2}\right) = \frac{16K^2}{L^2} - \frac{8\pi^3}{L^2}\frac{H}{L} \qquad (51)$$

This means that when r^2 is combined with a second order expression, the last term in r^2 will give a third order term, so in a second order theory we have

$$\frac{\partial\eta}{\partial x}\eta r^2 = \frac{\partial\eta}{\partial x}\eta\frac{16K^2}{L^2} \qquad (52)$$

η_c in eq. F3 can be expanded in powers of m to give

$$\eta_c = \frac{H}{2}\left(1+\frac{m}{8}+\cdots\right) \qquad (53)$$

This is used together with eqs. F1 and D4 in eq. F16 to give

$$\frac{\partial^2\eta}{\partial x^2} = -8\frac{H}{L^2}K^2\left[-1+2\,cn^2\theta\right] - 8\frac{H}{L^2}\pi^3\frac{H}{L}\left[1-4\,cn^2\theta+3\,cn^4\theta\right]$$

$$= -16\frac{K^2}{L^2}\eta + \text{second order terms} \qquad (54)$$

From this we get, with second order approximation, using eq. 52,

$$\frac{\partial\eta}{\partial x}\frac{\partial^2\eta}{\partial x^2} = -\frac{\partial\eta}{\partial x}\eta\frac{16K^2}{L^2} = -\frac{\partial\eta}{\partial x}\eta r^2 \qquad (55)$$

So for the next couple of pages it is important to remember from eq. D4 that mK^2 can be substituted by $\pi^3(H/L)$, which makes e.g. $(H/L)^2 mK^2$ of third order magnitude and thereby negligible.

By further differentiation we would find from mathematics

$$\frac{\partial^3\eta}{\partial x^3} = -\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}\left[1-2m+3m\,cn^2\theta\right] \qquad (56)$$

so that to the second order we have

$$\eta\frac{\partial^3\eta}{\partial x^3} = -\eta\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2} = -\eta\frac{\partial\eta}{\partial x}r^2 \qquad (57)$$

We then find with second order approximation using eqs. F1, F7, and 56

$$\frac{\partial^3\eta}{\partial x^3}+r6\eta\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}+\frac{\partial\eta}{\partial x}r^2=-\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}[1-2m+3m\,cn^2\theta]$$

$$+\frac{2\pi}{L}6H[cn^2\theta-\frac{1}{2}]\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}+\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}(1-\frac{m}{2})=0 \tag{58}$$

where in the second term η_c and r from eqs. F3 and F7 were expanded in series of m and then mK^2 was substituted by $\pi^3(H/L)$ through eq. D4, and third and higher order terms in H/L neglected.

We can consider r from eq. F7 a moment again,

$$r=\frac{4K}{L}\sqrt{1-\frac{m}{2}}=\frac{4K}{L}-\frac{mK}{L}+\cdots \tag{59}$$

With K from eq. 32 we get

$$r=\frac{2\pi}{L}+\frac{2\pi}{L}\left(\frac{m}{4}+\cdots\right)-\frac{mK}{L}+\cdots \tag{60}$$

This value for r was substituted into the second term in eq. 58. We then find, using c from eq. D10

$$g\frac{\partial\eta}{\partial x}+c^2\left[\frac{\partial^3\eta}{\partial x^3}\frac{1}{r}+6\eta\frac{\partial\eta}{\partial x}\frac{16K^2}{L^2}\right]=$$

$$g\left[\frac{\partial\eta}{\partial x}-\frac{1}{r}\left(\frac{1}{r}+\eta_c-\frac{H}{2}\right)\frac{\partial\eta}{\partial x}r^2\right]=0 \tag{61}$$

because $(\eta_c-\frac{H}{2})\frac{\partial\eta}{\partial x}r^2$ would give only third order terms as seen from eqs. 51 and 53. In the second term the coefficient of c^2 was substituted by $\frac{\partial\eta}{\partial x}r$, as found from the left side of eq. 58.

From eq. D14 we get, neglecting third and higher order terms

$$\frac{\partial p}{\partial x}=\gamma\frac{\partial\eta}{\partial x}+\rho c^2\left\{\left[\frac{\partial^3\eta}{\partial x^3}\frac{1}{r}-\left[5\frac{\partial\eta}{\partial x}\frac{\partial^2\eta}{\partial x^2}+\eta\frac{\partial^3\eta}{\partial x^3}\right]\right]\left[1-e^{r(z-y)}\right]\right.$$

$$\left.+\frac{\partial^2\eta}{\partial x^2}\frac{\partial\eta}{\partial x}e^{r(z-y)}+\frac{1}{2}\left[\frac{\partial\eta}{\partial x}\frac{\partial^2\eta}{\partial x^2}-\eta\frac{\partial^3\eta}{\partial x^3}\right]\left[1-e^{2r(z-y)}\right]\right\} \tag{62}$$

Eqs. D12 and D13 give, neglecting third and higher order terms

$$\frac{\partial u}{\partial t} = -c^2\left[\frac{\partial \eta}{\partial x} - \eta\frac{\partial \eta}{\partial x}r\right] r e^{r(z-y)} \tag{63}$$

$$u\frac{\partial u}{\partial x} = c^2 \eta\frac{\partial \eta}{\partial x} r^2 e^{2r(z-y)} \tag{64}$$

$$w\frac{\partial u}{\partial z} = -c^2 \eta\frac{\partial \eta}{\partial x} r^2 e^{2r(z-y)} \tag{65}$$

Eqs. 62, 63, 64, and 65 can be combined, and second order terms substituted by eqs. 55 and 57, so that we get

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{du}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} =$$

$$g\frac{\partial \eta}{\partial x} + c^2\left[\frac{\partial^3 \eta}{\partial x^3}\frac{1}{r} + 6\eta\frac{\partial \eta}{\partial x}\frac{16K^2}{L^2}\right]$$

$$-c^2\left[\frac{\partial^3 \eta}{\partial x^3} + r6\eta\frac{\partial \eta}{\partial x}\frac{16K^2}{L^2} + \frac{\partial \eta}{\partial x}r^2\right]\frac{1}{r}e^{r(z-y)} = 0 \tag{66}$$

where we used eqs. 61 and 58. Eq. 66 says that the horizontal dynamic equation is fulfilled to the second order of approximation.

ROTATION

The complete elliptic integral K can be expanded in powers of m as

$$K = K(m) = \frac{\pi}{2}\left[1 + \frac{m}{4} + \cdots\right] \tag{67}$$

We then get, using eq. D4

$$K^3 = \left[\frac{\pi}{2}\left[1 + \frac{m}{4} + \cdots\right]\right]^3 = \left(\frac{\pi}{2}\right)^3 + 3\frac{m}{4}K^2 + \cdots = \left(\frac{\pi}{2}\right)^3 + \frac{3}{4}\pi^3\frac{H}{L} + \cdots \tag{68}$$

From eqs. D12 and D13 we find the rotation

$$\Omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = c\left\{\eta r^2 + \frac{\partial^2 \eta}{\partial x^2} - \eta\frac{\partial^2 \eta}{\partial x^2}r - 2\left(\frac{\partial \eta}{\partial x}\right)^2 r\right\}e^{r(z-y)} \tag{69}$$

Using eqs. 53, 54, F7, and F15 we find

$$\Omega = c\{H(cn^2\theta - \frac{1}{2} + \frac{m}{8})\frac{16K^2}{L^2}(1-\frac{m}{2}) - \frac{8K^2H}{L^2}(m-1$$
$$-2(2m-1)cn^2\theta + 3m\,cn^4\theta) + \eta^2 16\frac{K^2}{L^2}r$$
$$+2\cdot 16\frac{K^2}{L^2}(\eta^2 - \frac{1}{4})r\}e^{r(z-y)} \tag{70}$$

Using eqs. D4 and 68 this will after a few calculations reduce to

$$\Omega = -c\frac{\pi^3 H^2}{L^3}e^{r(z-y)} \tag{71}$$

For an ideal fluid we must demand the rotation for a particle to be constant in time. This is also found by eq. 35 to the second order of approximation.

This rotation may not be as the wanted. Then the rotation can be changed to any second order magnitude with the same exponential distribution simply by using eq. D19 with the arbitrary constant δ instead of eq. D12. Such a change in u will not make any changes of second order magnitude in any of the previous considerations. To get an irrotational wave, e.g., we chose $\delta = 1/2$ in eq. D19. In chapter VI the description of waves with rotation of first order magnitude and with a more arbitrary distribution is included.