REGULAR WAVES

FOR PRACTICAL USE

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ABSTRACTS

This thesis considers two dimensional regular waves of finite amplitude.

The waves are developed directly from the equations of momentum and the continuity equation.

This makes it possible to find progressive and standing sinusoidal waves of first, second and higher orders. But it also makes it possible to find the cnoidal wave on infinite deep water or any arbitrary depth, a wave of interest for the engineer.

It is found essential for practical application of the final expressions that usual approximations are avoided in some of the boundary conditions for all types of waves here.

In chapter I the physical conditions for the motion of water in waves is considered from a practical point of view. It is concluded that usual hydrodynamic approximations may be accepted down in the fluid, but the boundary conditions should be fulfilled as close as possible in the final expressions ; like having no fluid pressure at the surface ; the surface particle at the wall in the standing wave should follow the surface ; etc.

In chapter II the most simple situation is considered : the progressive first order deep water wave. It is here tried to make a different wave theory that is in accordance with the considerations of chapter I . The development of the waves is found to be more simple and practical than usual for the engineer.

In chapter III a short historical background is given. The wave theories of this thesis have of course been made on the background of the good knowledge of waves obtained from the classical wave theories; despite that, the waves here are developed from the beginning so that they could have been given before the days of Airy.

In <u>chapter IV</u> the progressive first order wave on arbitrary depth is developed after the same basic principles so thoroughly discussed in chapter II. The surface profile will be the same as given by Airy, but velocities and pressure will be different.

In <u>chapter V</u> the standing first order wave on arbitrary depth is developed after the same basic principles. For the pressure on the vertical wall the difference in practical results between this theory and the classical theory is important.

In <u>chapter VI</u> the progressive deep water wave is considered again. Different types of second order and third order sinusoidal waves are developed. Arbitrary rotation of up to first order magnitude is also included. For irrotational motion the waves will be the well known Stokes' waves, except that those here fulfil boundary conditions better. The most important is the development of the deep water cnoidal wave.

In <u>chapter VII</u> sinusoidal second order waves on arbitrary depth is developed. The same basic theory gives both the progressive and the standing waves. The results may not always be of so much use for the engineer, but they are important for the considerations necessary to give the cnoidal wave in chapter IX.

In <u>chapter VIII</u> the shallow water cnoidal wave is developed employing the specific classical shallow water conditions. The wave profile is the same as the classical, but velocities and pressure are different. The results of this theory is of importance for the cnoidal wave of chapter IX.

In chapter IX the progressive finite amplitude wave of second order is considered again. This time the result is the cnoidal wave on arbitrary depth. This means that the expressions for the final formulas are the same for the wave on infinite deep water at one limit and for the solitary wave at another limit. After the study of wave theories in the preceding chapters, specially the deep water cnoidal wave of chapter VI, it should not be difficult to follow the theory here.

In chapter X the final expressions for the cnoidal wave on arbitrary depth are given. Further a wave table is given. This chapter is written with the intension that the engineer who only wants to do practical calculations with the new formulas only need to use this chapter for most purposes.

In chapter XI it is shown how the wave theory can be continued to give the third order progressive sinusoidal wave on arbitrary depth. This shows the third order celerity to depend on the wave height, a situation of interest for the cnoidal wave.

In chapter XII it is shown how the cnoidal theory also can be used on standing waves, by giving the standing deep water cnoidal waves. This means that the word 'cnoidal', which for 8o years was closely connected with the words progressive wave and shallow water has drifted all the way out to standing waves on infinite deep water.

Each chapter is made with the intension that it can be read independent of the other chapters. This means that a few lines are more or less repeated from chapter to chapter and that it is very seldom there is referred to equations in other chapters (chapters XI and XII are exceptions). So even chapter IX with the important cnoidal wave on arbitrary depth can in principle be read without reading the previous chapters. The first time it is though important to read the chapters from the beginning because many considerations on how to find the waves here are given together with the development of the more simple theories in the first chapters.

Each chapter gives the hydrodynamic development of the wave in the main part while all further considerations are given in appendices (with the exception of chapter X)

Just like the classical theories, the theories of this thesis are approximative, but at some important points the approximations are less, or even avoided.

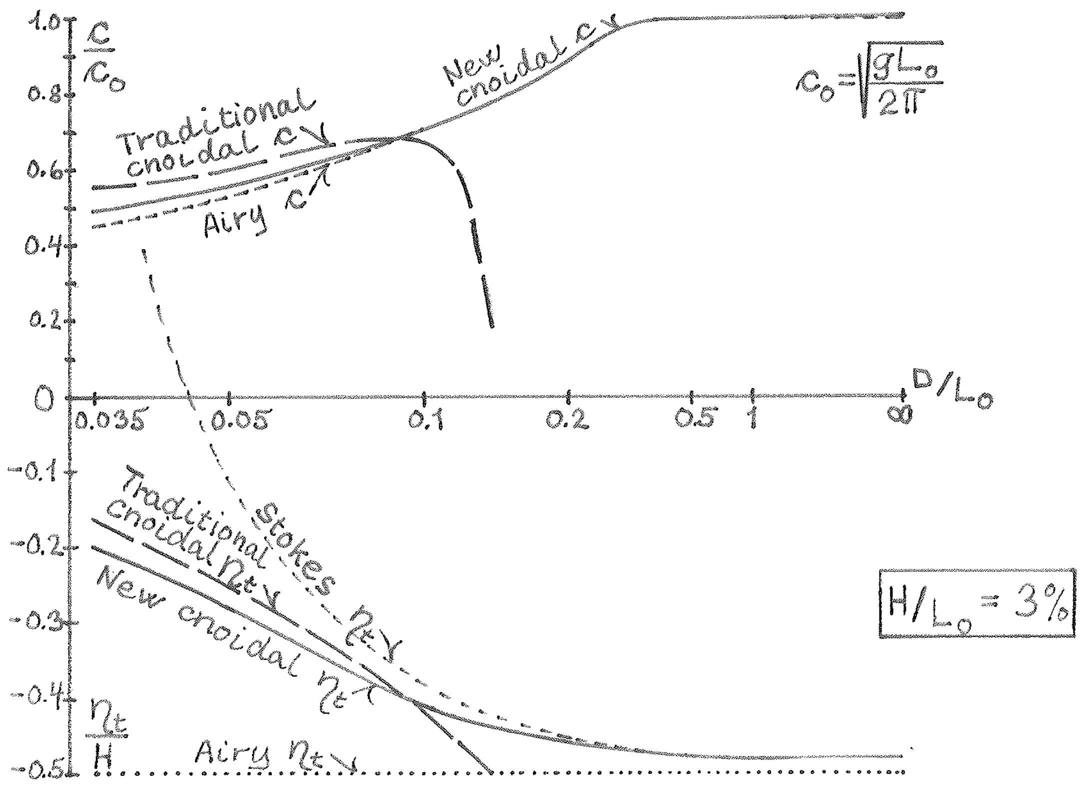

Fig.1. The celerity and the trough depth.
The traditional wave theories can only be used in specified regions, as shown, while the new cnoidal theory of chapter IX can be used on any water depth.

DANSK RESUMÉ

I et land som Danmark, der er omgivet af hav på alle sider, er der helt naturligt en stærk interesse for at kende de fysiske love i det vand, der slår ind imod vore kyster. Danskerne har da også fra før vikingerne haft mod til at forcere bølgerne for at nå over de store oceanografiske vidder.

Lige som interessen for bølger i forbindelse med anlæggelsen af Danmarks sidste store fiskerihavn på en stærkt udsat kyst var ved at lægge sig, kom de vældige (op til 25 m høje) nordsøbølger igen i søgelyset, på grund af den ivrige efterforskning efter olie i havbunden. Den fundne olie og gas skal så lagres og bringes i land og i alle tilfælde er der brug for kendskab til bølger.

Denne afhandling beskæftiger sig med regelmæssige bølger, samt hastigheder og tryk i bølger. Det er søgt at give de endelige formler en form, så de kan anvendes også på høje bølger. Derved opnås der f.eks en mere præcis beregning af det maksimale bølgetryk, hvilket kan give resultater, der er betydelig mindre end efter de beregningsmetoder, der hidtil er foreslået fra Danmarks Tekniske Højskole.

Desuden er der udviklet en ny hensigtsmæssig beskrivelse af høje bølger, med teorien for de såkaldte 'cnoidale' bølger på vilkårlig vanddybde. Hidtil har det været muligt at give en god beskrivelse af bølgeformen på dybt vand og på ret fladt vand, medens der har været vanskeligheder med vanddybden ind imellem, en dybde som ofte er foretrukket til havne. Den nye bølgebeskivelse kan benyttes overalt med de samme formeludtryk.

I kapitel I betragtes bølgen ud fra et ingeniørmæssigt synspunkt, hvilket afslører nogle pudsigheder i de traditionelle formler, samt stiller nogle krav, der ønskes opfyldt.

I kapitel II findes den regelmæssige fremadskridende dybvandsbølge af første orden på en måde så de vanskeligheder, der blev konstateret i kapitel I bliver løst.

I kapitel III gives der en kort historisk baggrund.

I kapitel IV udvides bølgeteorien fra kapitel II til at gælde vilkårlig vanddybde.

I kapitel V findes den regelmæssige stående bølge af første orden, med speciel opmærksomhed rettet mod bølgetrykket på den lodrette væg.

I kapitel VI findes anden og højere ordens dybvandsbølgen, hvilket også fører til den cnoidale dybvandsbølge.

I kapitel VII udvides teorien fra kapitel V til at inkludere anden ordens fremadskridende og stående bølger.

I kapitel VIII udledes den cnoidale fladvandsbølge, hvis overfladeform har været kendt længe.

I kapitel IX findes så den længe savnede cnoidale bølge på vilkårlig vanddybde.

I kapitel X gives en oversigt over de fleste af de formler, som har interesse for praktikeren, samt en bølgetabel over den nye cnoidale bølge.

I kapitel XI udvides teorien fra kapitel VII til at give tredie ordens fremadskridende bølger.

I kapitel XII afsluttes med den stående cnoidale dybvandsbølge.

De fundne overfladeformer for sinusbølgerne og den cnoidale fladvandsbølge er de allerede velkendte, medens udtrykkene for partikelhastigheder og tryk er nye. I teorien for den cnoidale bølge på vilkårlig vanddybde, som almindeligvis bør anvendes for anden ordens bølger, er alt nyt. Det samme er tilfældet for den cnoidale dybvandsbølge.

INTRODUCTION

In a highly developed technical and specialized world the need for transportation is as big as ever. Any small industrial product (e.g. a ball point pen) is usually composed of materials that originate from very different places in the world. During the fabrication process the different parts of the final product may be sent back and forth around the world, to find the place where each part can be made at as low costs as possible.

Transportation over land, and specially by air is getting increasing attension, because it is fast and it is convenient. But the most important form of transportation is by ship, across the oceans. In transportation of heavy goods over a longer distance the ocean fares are unchallenged. The low shipping rates are important for the growing material wealth of the world as a whole, (but may also cause parts of the world to be retarded in the industrial development, because of superior competition from far away). The shipping rates could though in many cases be much lower. This is felt by many engineers that have been involved in engineering projects far away (like for the author the design and administration of the construction of a modern school in the middle of Africa from a Copenhagen office, and in which case much of the material needed, best could be shipped down from Europe). It is not the thousands of kilometres on the ocean that cost. The problems are on the land side.

There is a big need all over the world for better harbours in better contact with the land transportation system. The natural harbours in sheltered areas are in many cases not sufficient any more, and on the land side they are often overcrowded with streets and buildings that are irrelevant for the business of the harbour. So new harbours are needed outside the big cities. This means that it can be necessary to build harbours on coasts that are more exposed to wave action.

With the high developed technology and with the increasing demand for raw materials the attention has been turned to the floor of the oceans. Oil and gas explorations are taking place in the North Sea with 25 m high waves, among the icebergs around Greenland, and all over the world. And after the oil and the minerals have been brought up from below the sea bottom they have to be brought to land. So there is a big need for a good knowledge of the action of the ocean.

When an ingeneer is designing harbours and other constructions at the coast, or off shore, there are a number of factors involved. Besides the problems, known from designing constructions on land, and from hydraulic constructions in e.g. rivers, the design of coastal and off shore structures must first of all take into consideration the wave action. What is of interest is then to find the action from the natural waves. These waves are however rather complicated to describe in detail. A necessary background is then first of all to study the regular waves, to get practical reasonable results from these more simple waves, and to get a good understanding of the physical processes in the wave.

So this thesis is confined to the study of two-dimensional, regular gravity waves, and their interaction with the vertical wall.

CLOSING REMARKS

The author feels that the 'cnoidal' functions are unchallenged in describing regular gravity surface waves. In this thesis they are used with the shown advantage to describe the second order wave, resulting in just one expression, $\eta = H\,cn^2\theta$, for any water depth.

The same short expression can be used also for higher order waves. The higher order improvements will then result in a better determination of the parameter m, maybe to the extent of letting m be a variable, in the same way as we found for m for the standing cnoidal wave of chapter XII. Considering e.g. the long expressions of the third order sinusoidal theory of chapter XI it will be nice to get the above short expression.

But even when not going so far as to make higher order theories, there are many possibilities to improve the theories, we already have, as shown in chapters I - V for the first order theory, and later for the cnoidal theory. If an expression is found not to fulfil a boundary condition as good as wanted, the necessary correction in the expression can be made with the right higher order terms. This can be kept in mind if an expression from the new cnoidal theory for some specific purpose should show not to be so good.

We could also use this procedure to make more radical changes in the whole cnoidal theory. In our theory here we have actually mathematically used mK^2 or m as a governing expression, so that higher order terms in m were negligible. This means that in principle higher order terms in m can be included or dropped as wanted. Then it could be of interest also to include considerations from the mathematical 'other end' using the complementary modulus, or $m_c = 1-m$, which physically means that during the development of the wave theory higher order terms are not only substituted by first order sinusoidal expressions, but also by solitary wave expressions (as partly done in chapter IX for R).

So there are many possibilities with this cnoidal theory.

PROPOSALS FOR FURTHER RESEARCH

It is felt that the wave theories of this thesis can be used for further research along several lines :

The scientist in wave hydrodynamics with an interest in the cnoidal wave on arbitrary depth of chapter IX might like to have the third order cnoidal wave. This is of interest not 'just to get an order more' but to have the shallow water limit and the celerity better determined.

With an interest in waves on currents it may be of interest to expand the considerations of rotational waves.

Although the results of chapter V can be used to an approximative calculation of the wave pressure on a vertical wall from cnoidal waves, it will be of interest to continue the theory of chapters VII and XII to find the standing cnoidal wave on arbitrary depth (which may not be a 'normal' standing wave).

It should also be possible to extend the theory here to find the waves of three dimensions, when the three dimensional situation of interest is specified.

Further it can be of interest to investigate waves on water with an inclined bottom, eventually combined vith the above mentioned situations.

There are of course many more problems in wave hydraulics, but they have not been considered by the author.

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